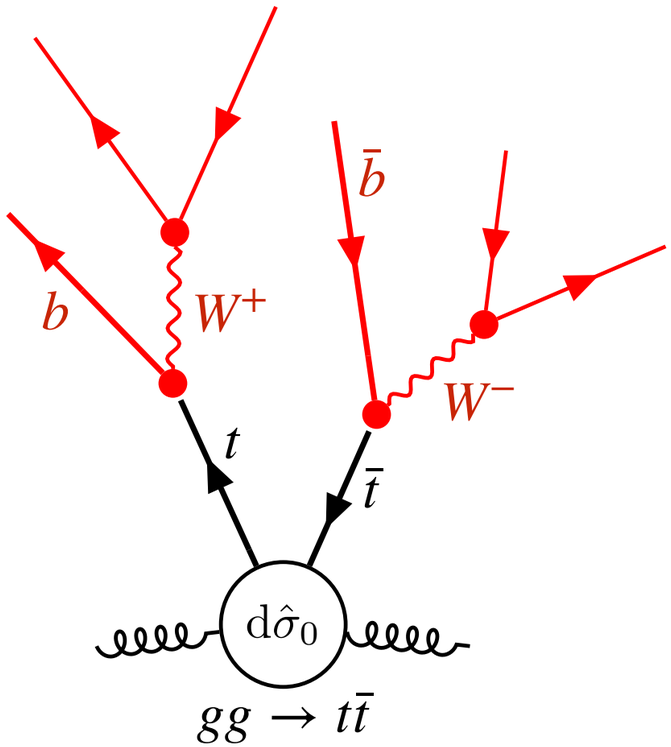
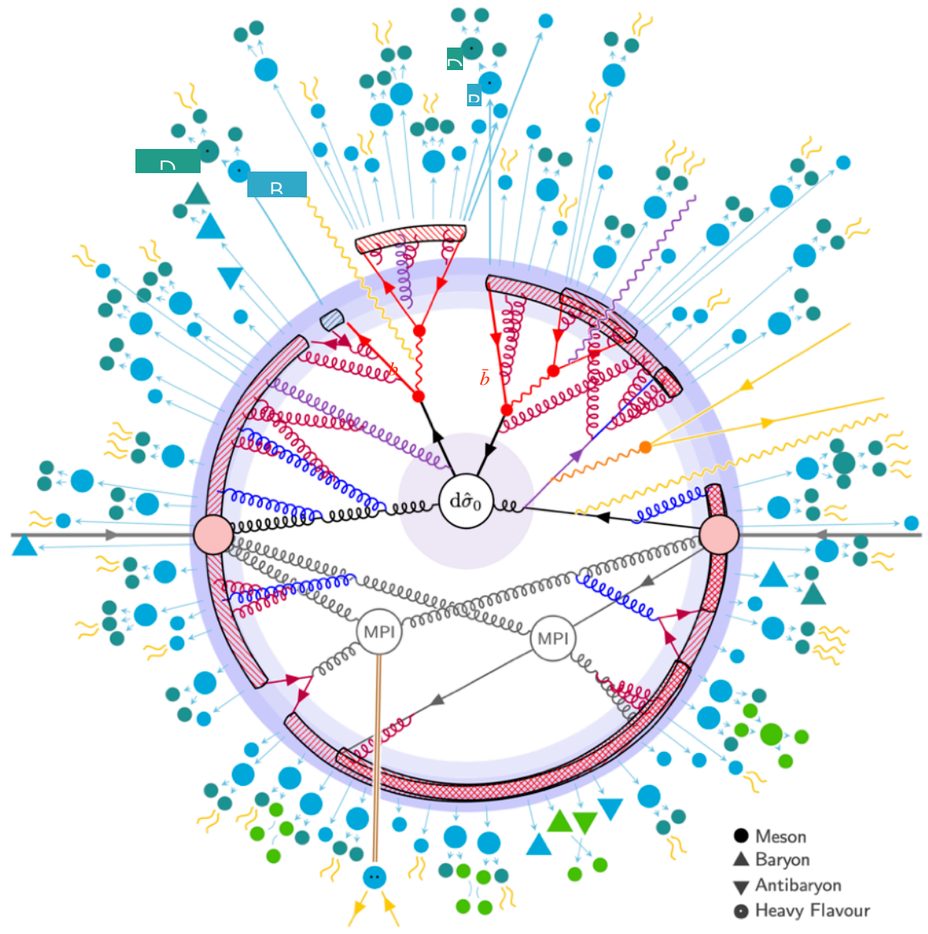
Jet Substructure from Theory to Experiment

— via Monte Carlo Event Generators





Related to:
PS 2609.18102,
Helliwell, Scyboz, **PS** 2512.07370,
El-Menoufi, Preuss, Scyboz, **PS** 2412.14242

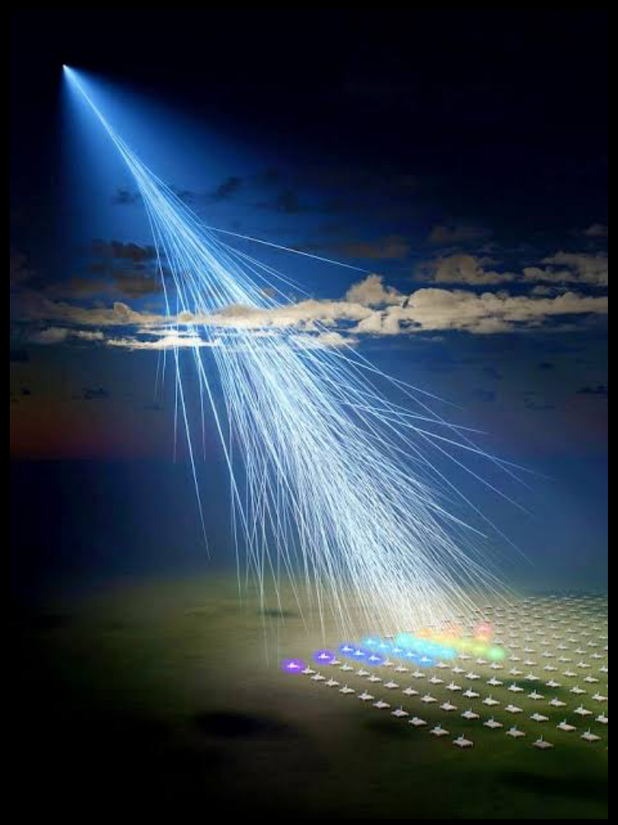
Peter Skands (Monash U)

MONASH University
MONASH WARWICK ALLIANCE
Australian Government
Australian Research Council
MCnet

This talk is about bremsstrahlung

Radiation from accelerated charges

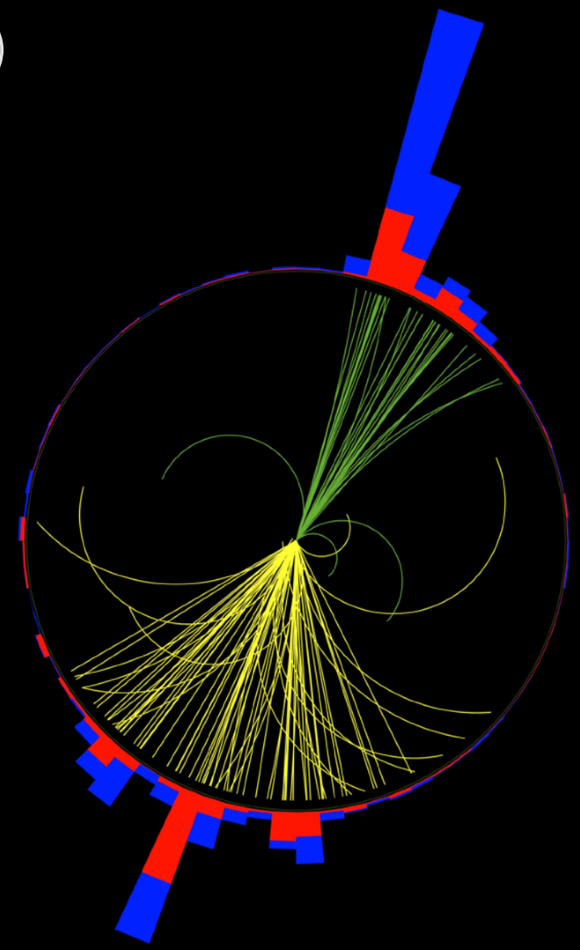
Ubiquitous across physics (& astrophysics)



EM component of cosmic-ray air showers. Predominantly from $e^{\pm} \to e^{\pm}\gamma$, $\gamma \to e^{+}e^{-}$, etc.

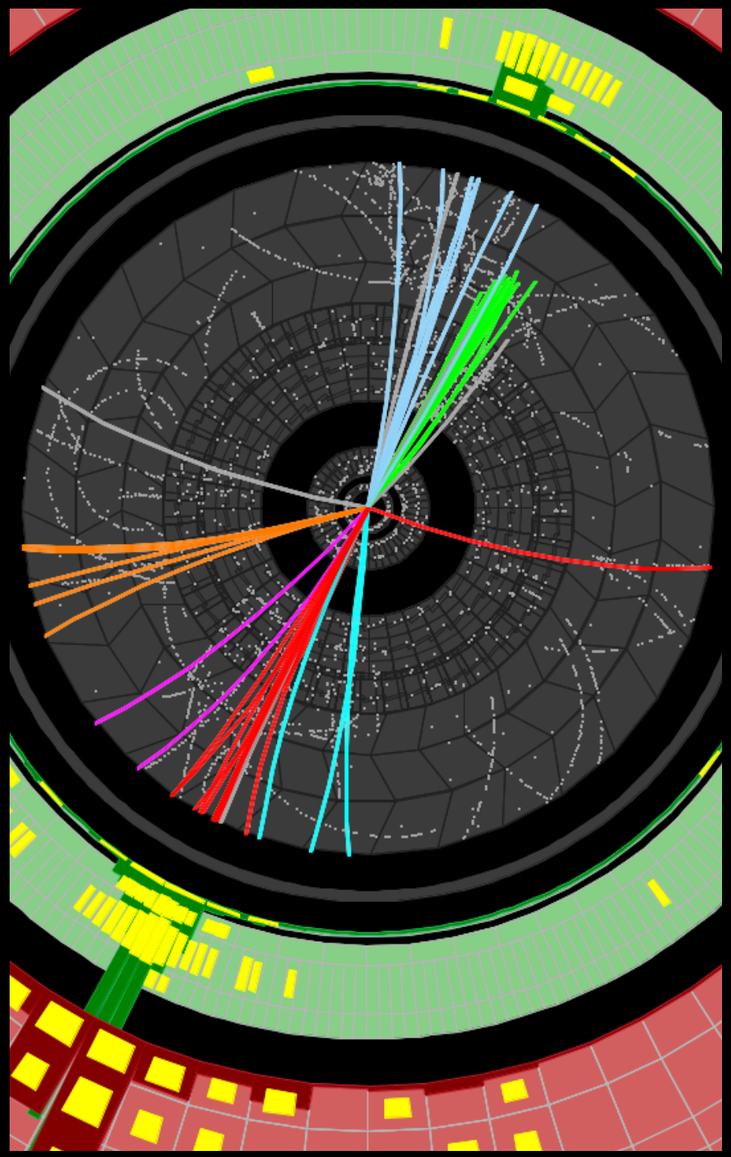


Synchrotron Radiation (BNL NS-LSII)



This talk: Jets of QCD bremsstrahlung in high-energy particle collisions

Theory ⟷ Experiment



Multiple jets in the ATLAS experiment

Accurate theoretical description of jets ⟷ good experimental control of systematics

Important for jet energy scale calibrations (JES)

Vital for precision measurements involving jets

+ Jet substructure is a subfield in its own right

E.g.: **searches** for (boosted) New-Physics particles decaying to jets ➜ non-QCD-like substructure

Quark vs gluon jet discrimination, flavour tagging

Jet "quenching" in quark-gluon plasma

Jet substructure ≈ physics of bremsstrahlung

Correspondence: leading behaviour is classical

≈ Multipole expansion + quantum corrections

Radiation from an accelerated charge

High-frequency end will depend on what exactly happened here.

Charge in uniform motion, from $t \to -\infty$

p

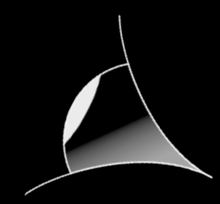
POW!

p'

Scattered charge with new direction. In uniform motion, to $t \to +\infty$

Sudden kick at $t = 0$, with characteristic momentum transfer Q

k



Observer

Sees a disturbance in the EM fields.

Represent charge as current density, solve (F.T. of) Maxwell eq $\partial^2 A^\mu = j^\mu$

$\Rightarrow$ the characteristic spectrum of (classical) **bremsstrahlung**

Energy density $\frac{1}{2}\left(|\vec{E}|^2 + |\vec{B}|^2\right) \mathrm{d}x^3$

Fourier Transform $= \frac{e^2}{2}\left[\frac{2(p\cdot p')}{(p\cdot k)(p'\cdot k)}\right]\frac{\mathrm{d}^3 k}{(2\pi)^3}$

The "**eikonal**" or "**dipole**" factor

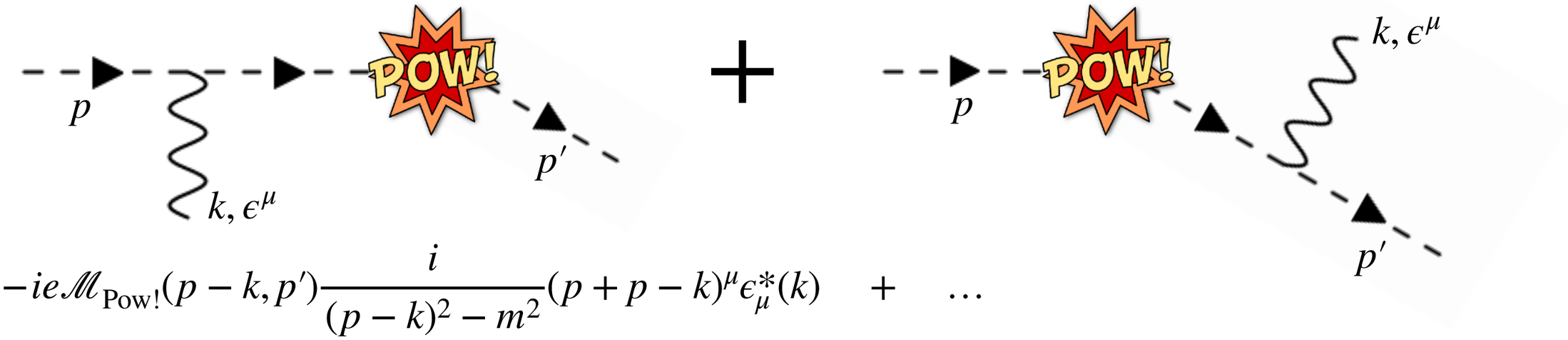
Singular for $\omega_k \to 0$ (**soft**)

And for $\theta_{kp} \vee \theta_{kp'} \to 0$ (**collinear**)

Quantum treatment

First-order QED correction to scattering of a charged particle

POW! = some operator that annihilates a particle with p and creates one with p'



$$-ie\mathscr{M}_{\text{Pow!}}(p-k,p')\frac{i}{(p-k)^2-m^2}(p+p-k)^\mu\epsilon^*_\mu(k) \quad + \quad \ldots$$

Sum, square, and sum over photon polarisations

Total probability density per phase-space element:

$$\frac{\mathrm{d}\sigma(p\to p'+k)}{\mathrm{d}\sigma(p\to p')\,\mathrm{d}^3k} = e^2\left(\frac{2(p\cdot p')}{(p\cdot k)(p'\cdot k)} + \textbf{mass, spin, and Pow! dependence}\right)$$

Same as classical

High-frequency end depends on what "Pow!" is

The context: high-energy particle collisions

Schematic of an LHC proton-proton collision (simplified)

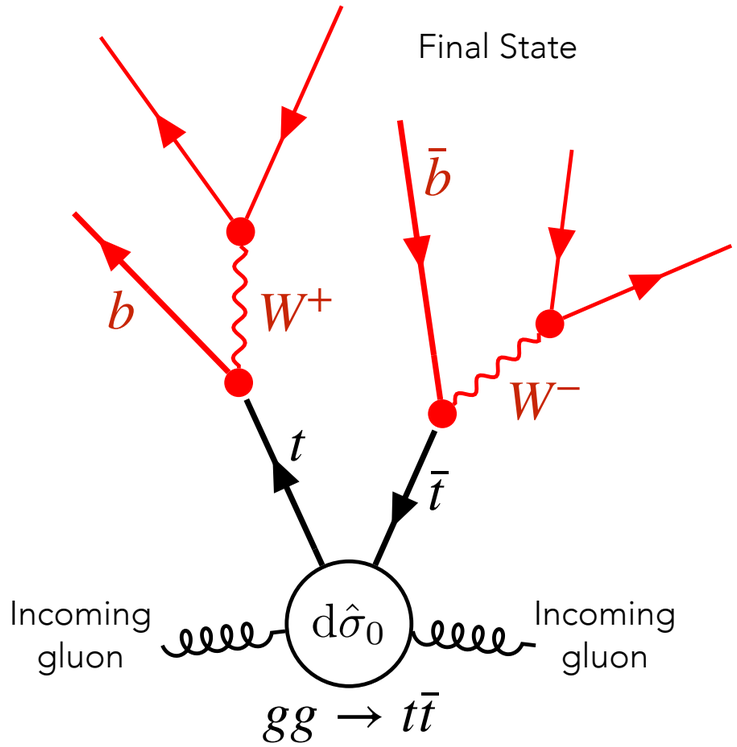
Hard Processes

○ Hard Interaction

● Resonance Decays

"Hard" means large momentum transfers ≫ 1 GeV

= the "sudden kicks" that define the initial "accelerated charges"



(: incoming lines are crossed)*

[Figure from Bierlich et al., SciPost Phys.Codeb. (2022) 8 arXiv:2203.11601]

Bremsstrahlung

Schematic of an LHC proton-proton collision (simplified)

Hard Processes
- ◯ Hard Interaction
- ● Resonance Decays

"Hard" means large momentum transfers ≫ 1 GeV

Brems-strahlung
- ■ Final-State Radiation
- ■ Initial-State Radiation*

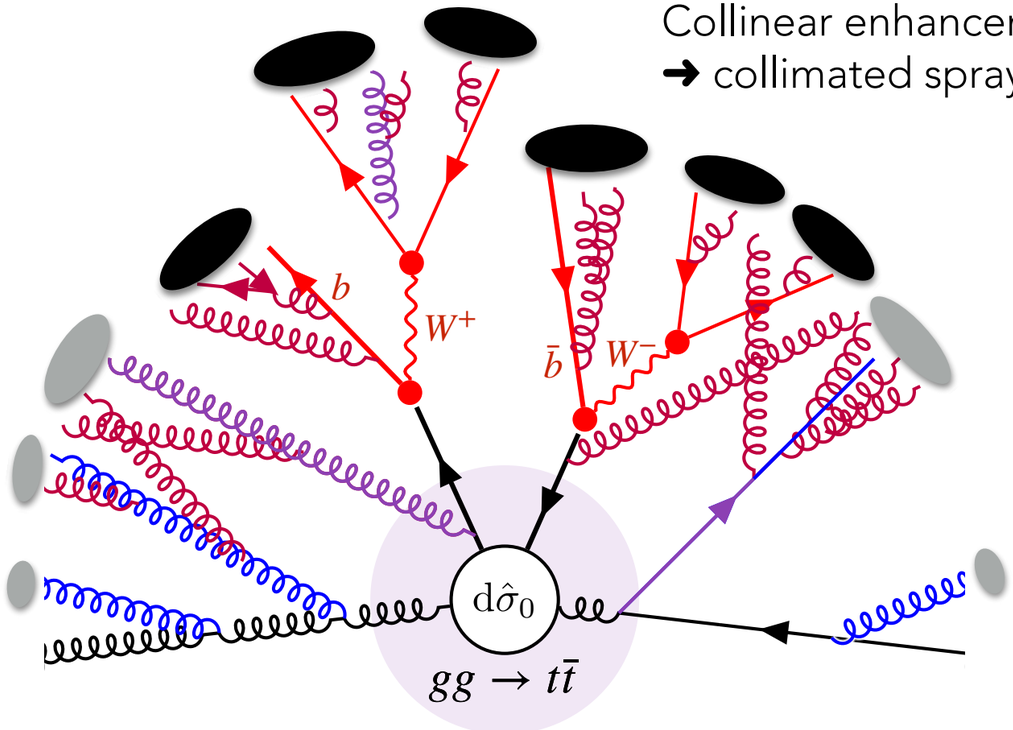
Down to momentum transfers ∼ 1 GeV

Bremsstrahlung cascades in QCD

$$q \to qg,\ g \to q\bar{q},\ g \to gg$$

➜ The "hard" partons turn into **"jets"** of partons (quarks and gluons)

(+ EM not forgotten: $q \to q\gamma,\ \gamma \to f\bar{f}$)



Collinear enhancements
➜ collimated sprays: **jets**

Partonic jets initiated by **hard** partons with substructure generated by bremsstrahlung

Additional jets generated purely by **bremsstrahlung**

(: incoming lines are crossed)*

[Figure from Bierlich et al., SciPost Phys.Codeb. (2022) 8 arXiv:2203.11601]

+ **Confinement** (not covered today)

Schematic of an LHC proton-proton collision (still simplified)

Hard Processes
- ◯ Hard Interaction
- ● Resonance Decays

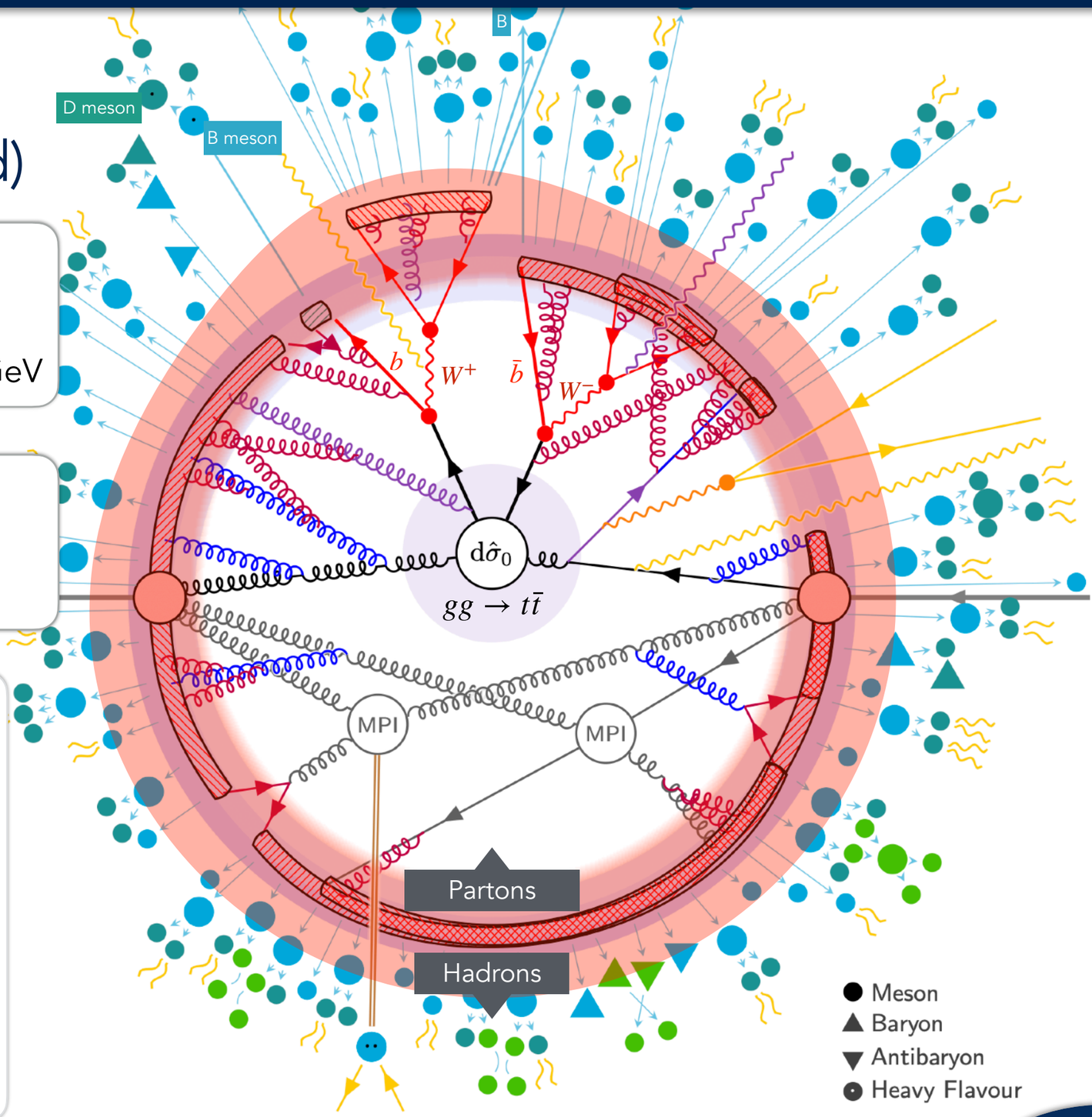
"Hard" means large momentum transfers ≫ 1 GeV

Brems-strahlung
- ■ Final-State Radiation
- ■ Initial-State Radiation*

Down to momentum transfers ∼ 1 GeV

Hadron-ization
- ◯ Multiparton Interactions
- □ Beam Remnants*
- ▨ Strings
- ▨ Ministrings / Clusters
- Colour Reconnections
- String Interactions
- Bose-Einstein & Fermi-Dirac
- Primary Hadrons
- Secondary Hadrons

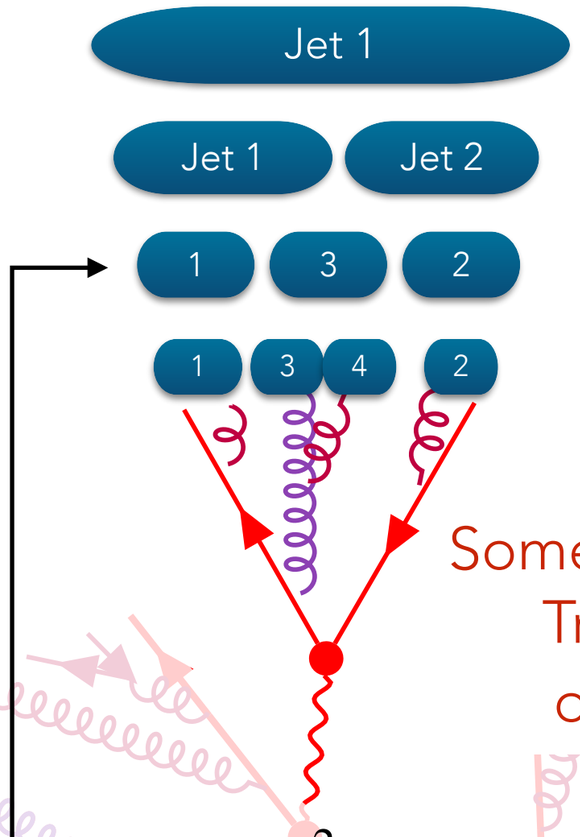
Momentum transfers $\lesssim 500\,\mathrm{MeV}$



What do we actually measure?

Jet clustering algorithms

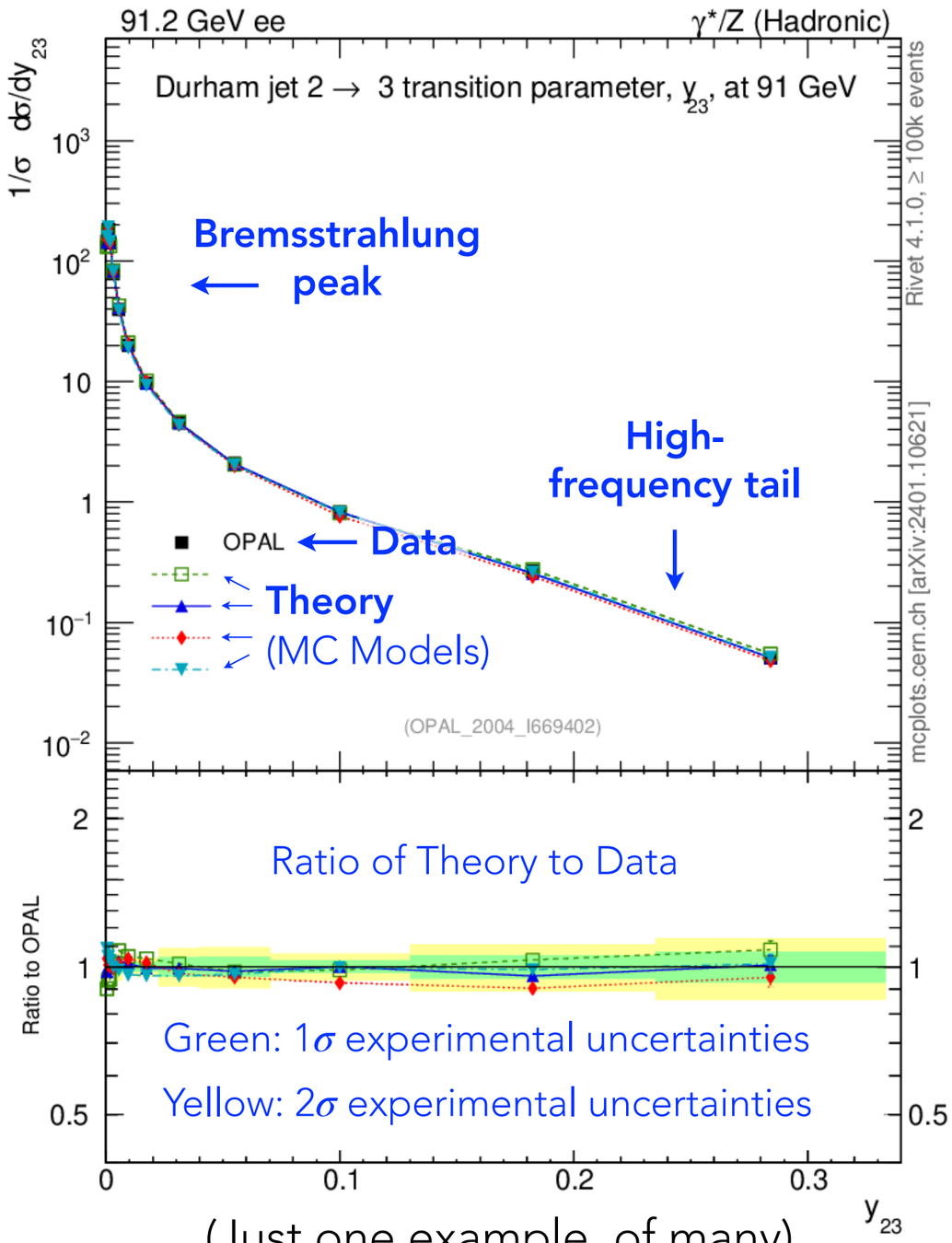
Look at events at different scales



Substructure resolution scale, Q_S

Some measure of **"hardness"**: Transverse momentum, opening angle, ΔR, …

$y_{23} = \frac{Q_3^2}{E_{\mathrm{CM}}^2}$: hardness scale at which the 3rd jet becomes resolved

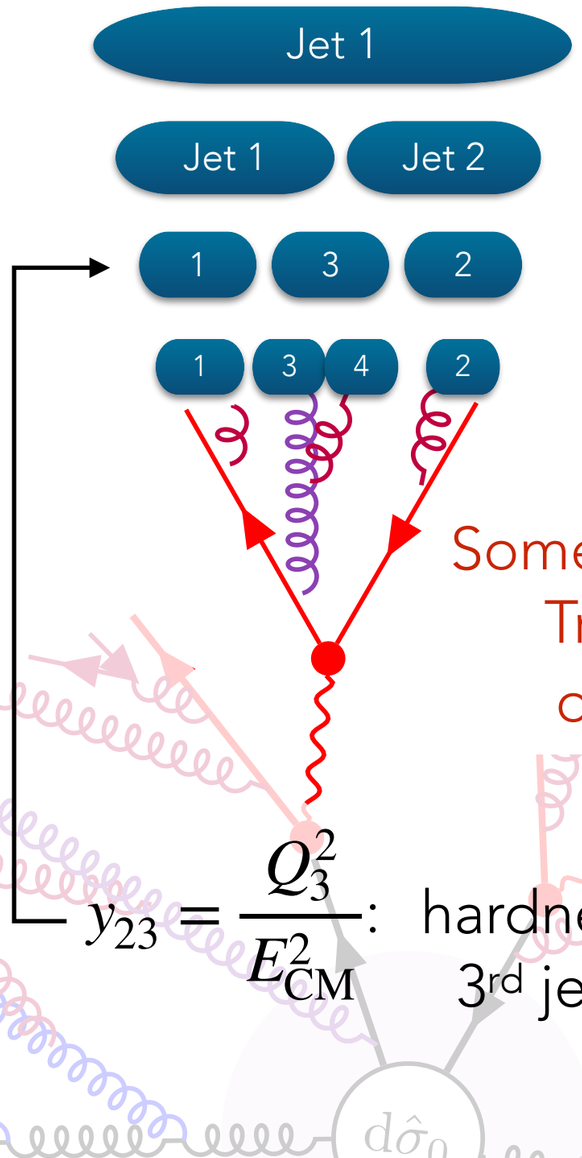


(Just one example, of many)

To look at the brems peak, change to log x axis

Jet clustering algorithms

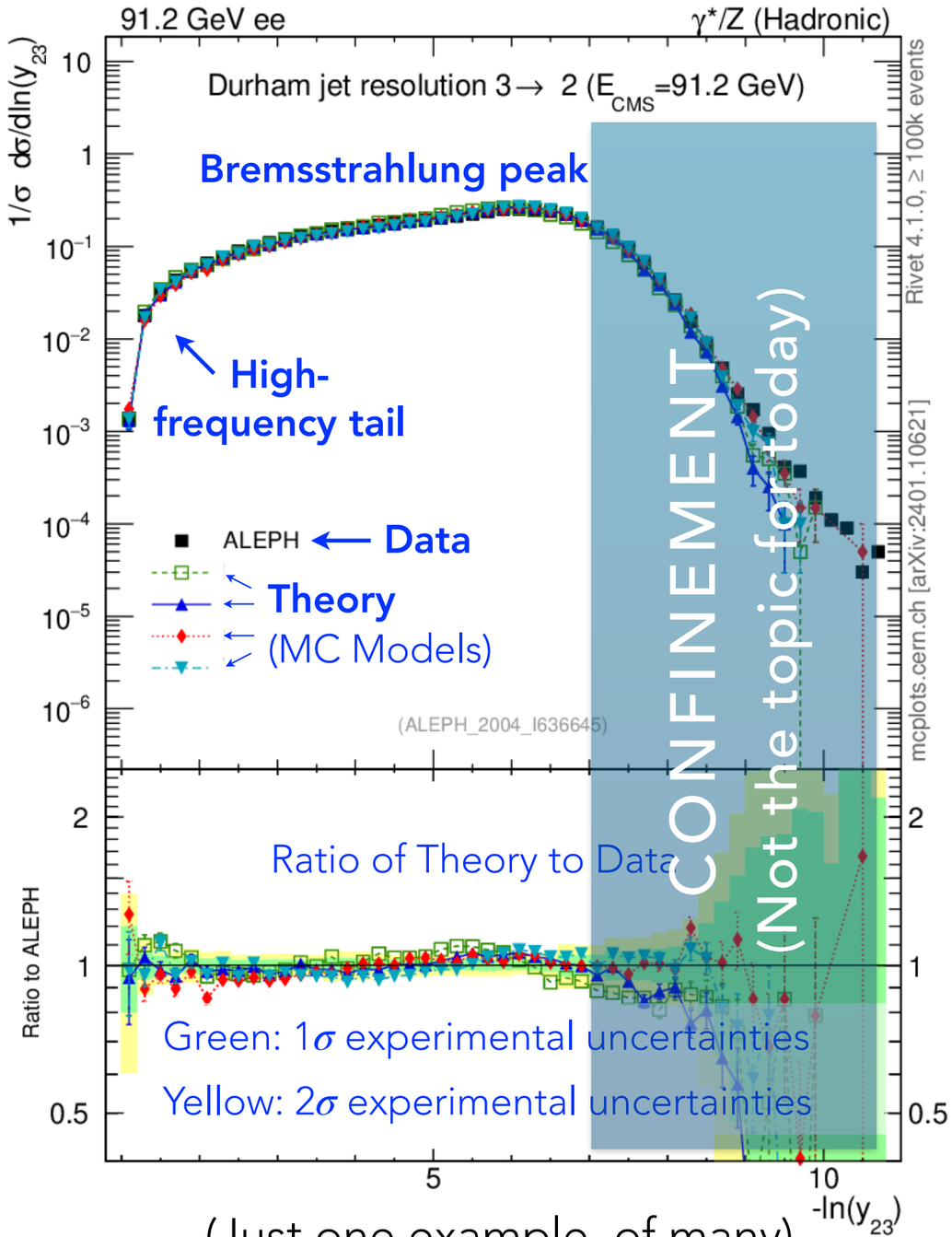
Look at events at different scales



Substructure resolution scale, Q_S

Some measure of **"hardness"**: Transverse momentum, opening angle, ΔR, …

$y_{23} = \frac{Q_3^2}{E_{CM}^2}$: hardness scale at which the 3rd jet becomes resolved



(Just one example, of many)

To look at the brems peak, change to log x axis

Theory models look pretty good?

A priori, a little surprising.

They employ significant **approximations**:

- **Leading Order (LO)***

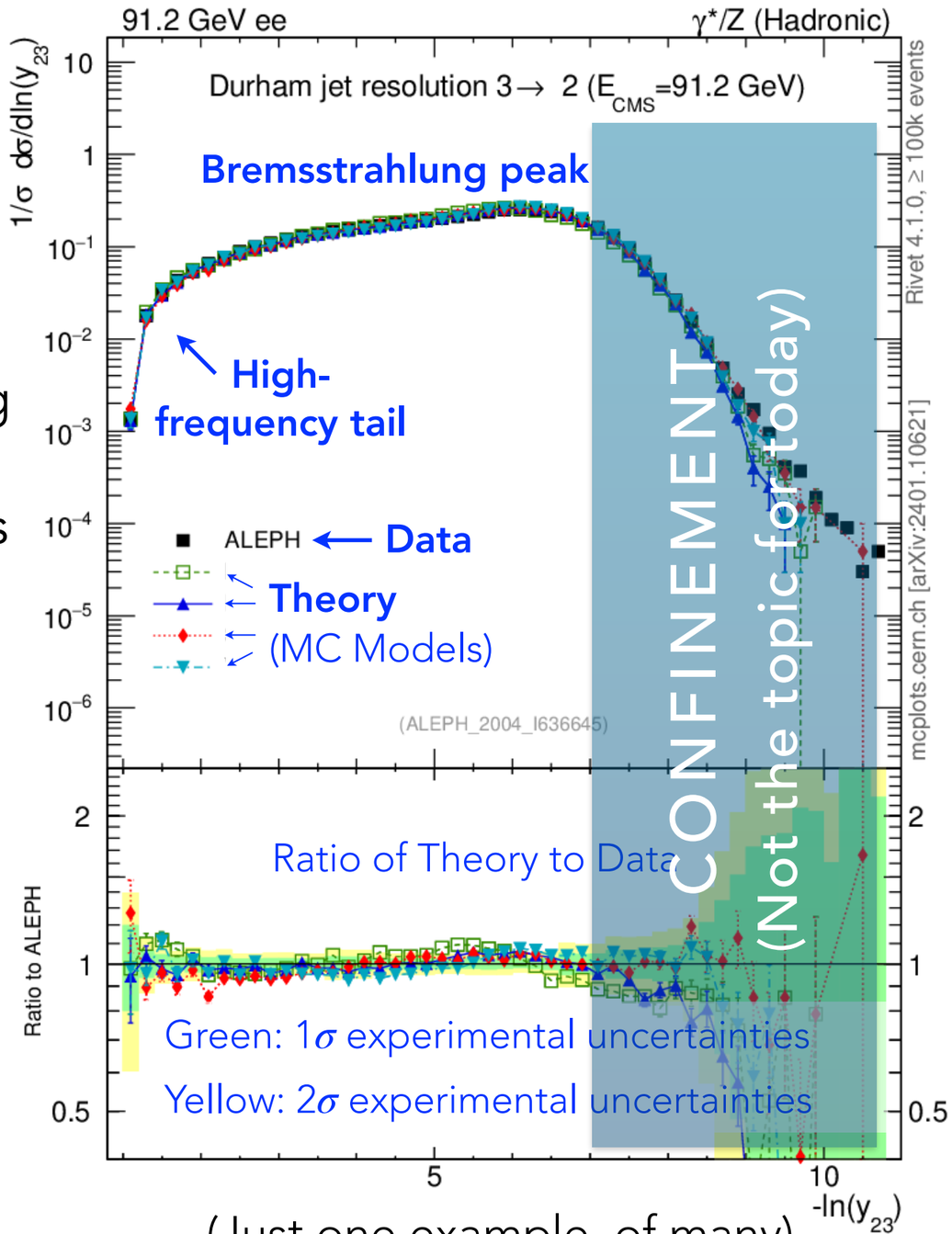
  Significant progress in last ~ 20 years on matching to fixed-order perturbation theory (FOPT)
  (Matching & Merging) ➜ (N)NLO & LO corrections
  Most important for high-freq. tail **➜ extra jets**
  Not ideally suited for jet **substructure**

- **Leading Color (LC)*** (Known to be quite good)

- **Leading Logarithmic (LL)***

  For the past 40 years, this was ~ all we could say
  But LL really isn't very good

* = + improvements for some aspects / in some limits



(Just one example, of many)

To define “LL” ➜ Define an observable

Not all observables are equal. I only show subjet p_T spectra.

Keep in mind:

Angular or energy spectra not identical to p_T spectra.

Further relevant properties of observables:

Inclusive vs exclusive sub-jet rates, azimuthally averaged vs not, hemisphere vs global, single-prong vs multi-prong, effects of filtering and grooming, …

Not to mention 27-dim correlations picked up by Machine-Learning jet taggers

Used to “tag" properties of jets, original quark (or g) flavor, QCD vs non-QCD, …

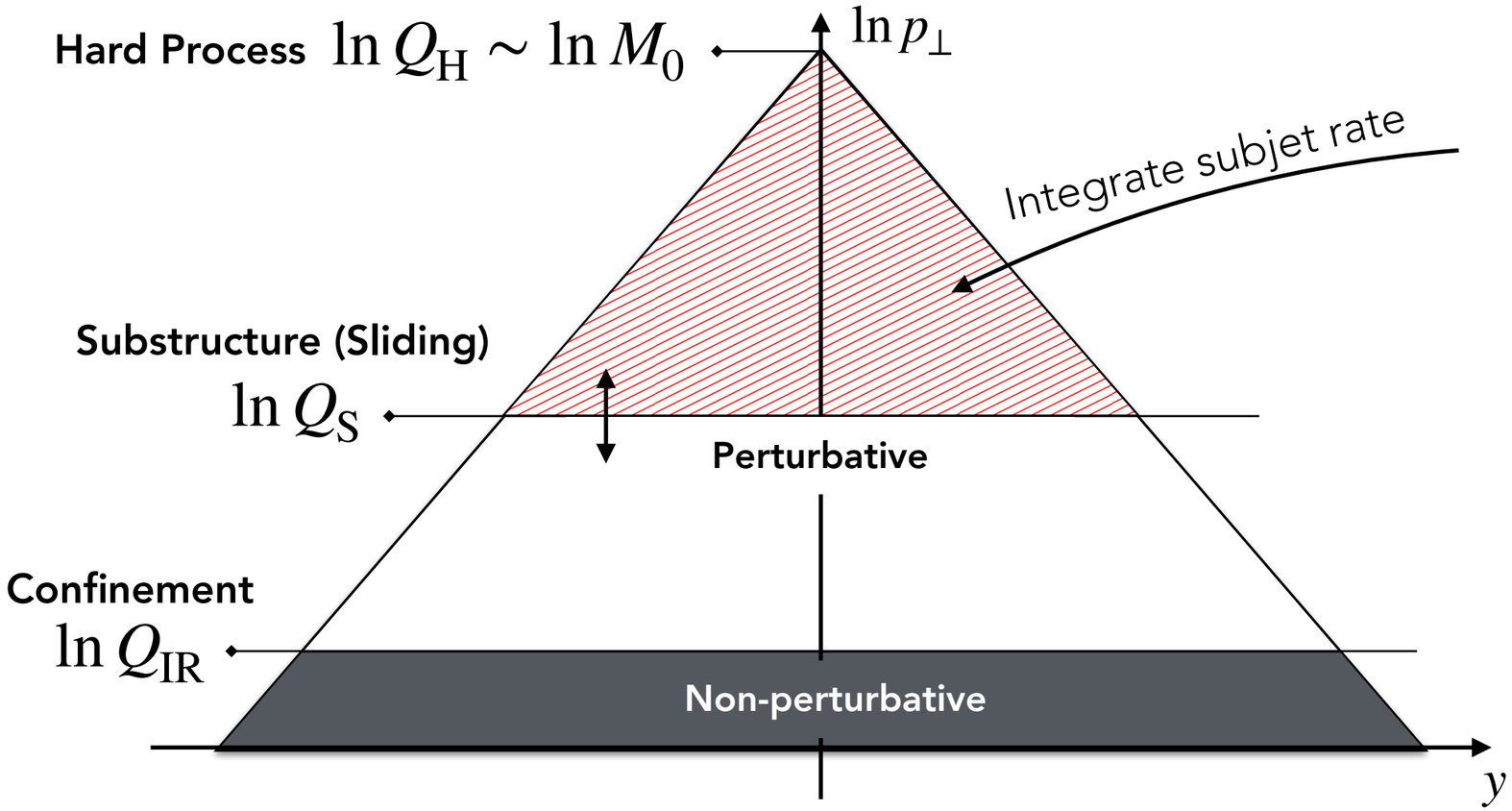
Have to start somewhere.

Keep in mind this is just a starting point.

Note also: for **QCD,** should also show expansion in powers of $1/N_C^2$
For simplicity + to avoid clutter: for now, keep *implicit*: $1/N_C^2 \leftrightarrow \alpha_s \sim \mathcal{O}(0.1)$

What is LL? Integrating the eikonal ➜ Logarithms

For a hard process, X, consider $\mathrm{d}\sigma_{\text{substructure}}$ for a $p_\perp$-like (jet) resolution



Generic Radiation Phase Space

"Lund Plane"

In log-log variables: $\ln(p_\perp)$ and rapidity, $y = \ln\left(\frac{E + p_{||}}{p_\perp^2}\right)$

$$2\pi\alpha_s C_F \int_{Q_S}^{Q_H} \frac{M_0^2}{(p\cdot k)(p'\cdot k)} \mathrm{d}\Phi_1$$

$$= \frac{2\alpha_s C_F}{\pi} \int_{Q_S}^{Q_H} \mathrm{d}\ln(p_\perp)\,\mathrm{d}y\,\frac{\mathrm{d}\phi}{2\pi}$$

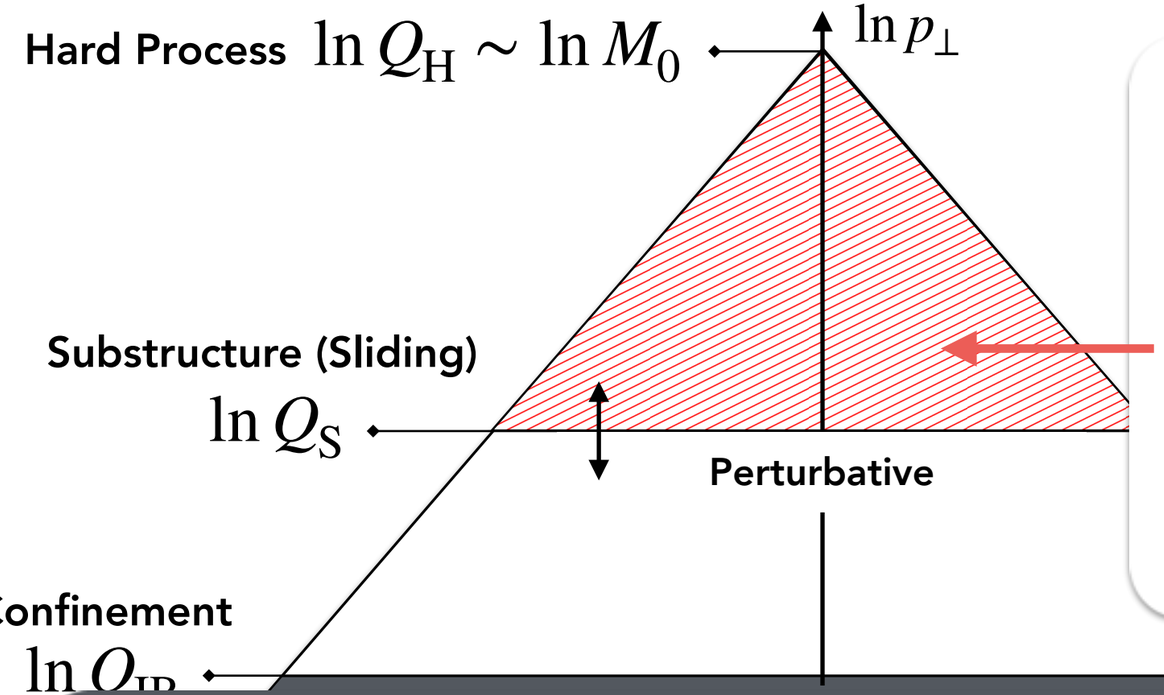
$$= \frac{2\alpha_s C_F}{\pi} \ln^2\left(\frac{Q_H}{Q_S}\right) + \ldots$$

$$\approx 0.81\,\alpha_s L^2$$

"LL" is basically exponentiating this + running coupling

QCD Perturbation Theory

For a hard process, X, consider $\mathrm{d}\sigma_{\mathrm{substructure}}$ for a p_T-like (jet) resolution



$$\int_{Q_S}^{Q_H} \mathrm{d}\sigma_{X+\mathrm{jet}}^{\mathrm{inclusive}} \propto \alpha_s(1 + L + L^2) + \alpha_s^2(1 + \ldots + L^4) + \alpha_s^3(1 + \ldots + L^6) + \ldots$$

$$L = \ln(Q_H/Q_S)$$

For simple observables, the logarithmic coefficients "exponentiate":

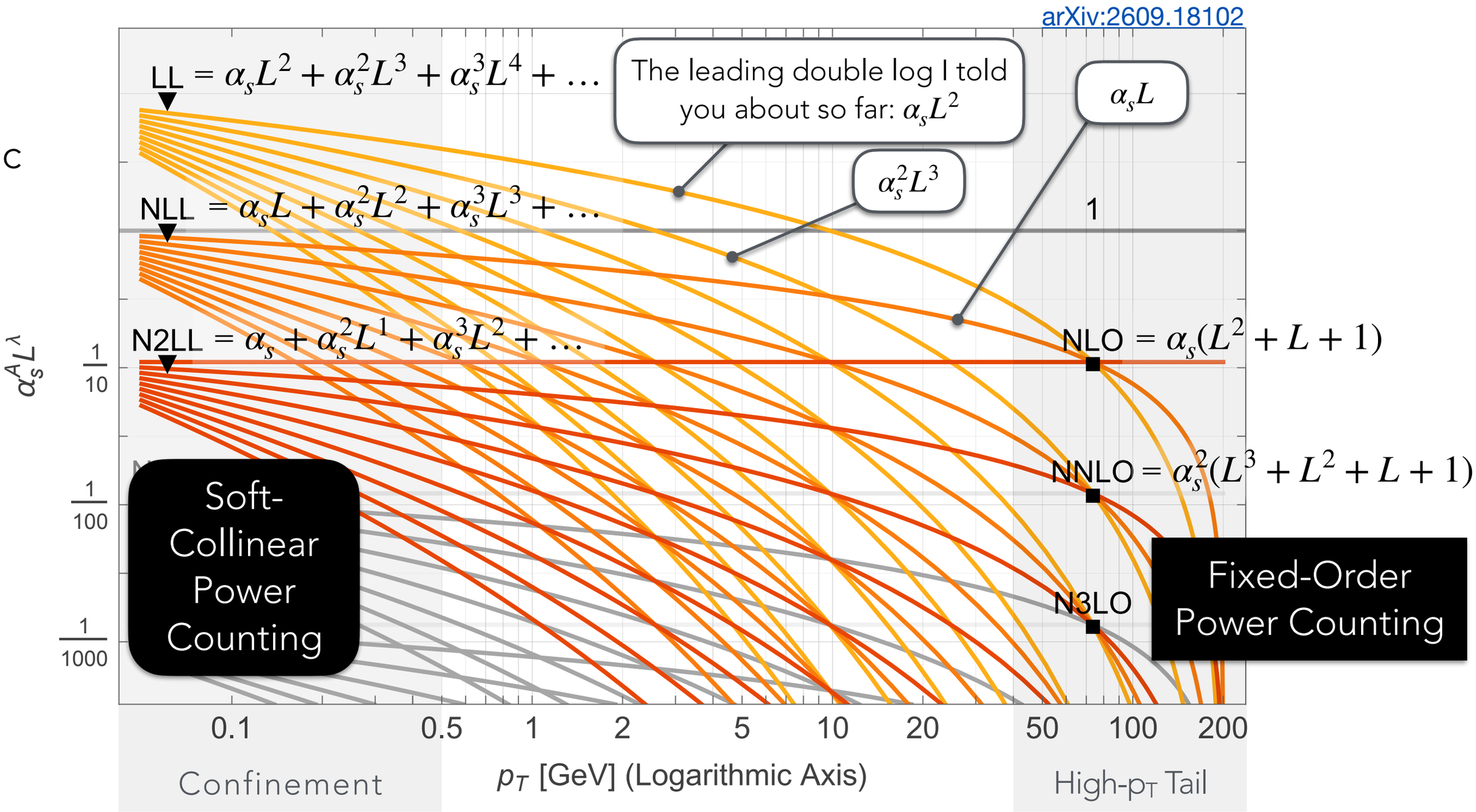
$$\sigma_X^{\mathrm{incl}}(1 + \alpha_s + \alpha_s^2 + \ldots)\exp\left(-\sum_{A=1}^{\infty}\alpha_s^A\sum_{\lambda=1}^{A+1}L^\lambda\right)$$

I'll focus on this series

"Cat's-Cradle Diagram" of Perturbative Coefficients

Hard Scale $Q_H = 200\,\text{GeV}$, $\alpha_s(Q_H) = 0.11$, $L \equiv \ln(Q_H/p_T)$

Plot each generic term $\alpha_s^A L^\lambda$

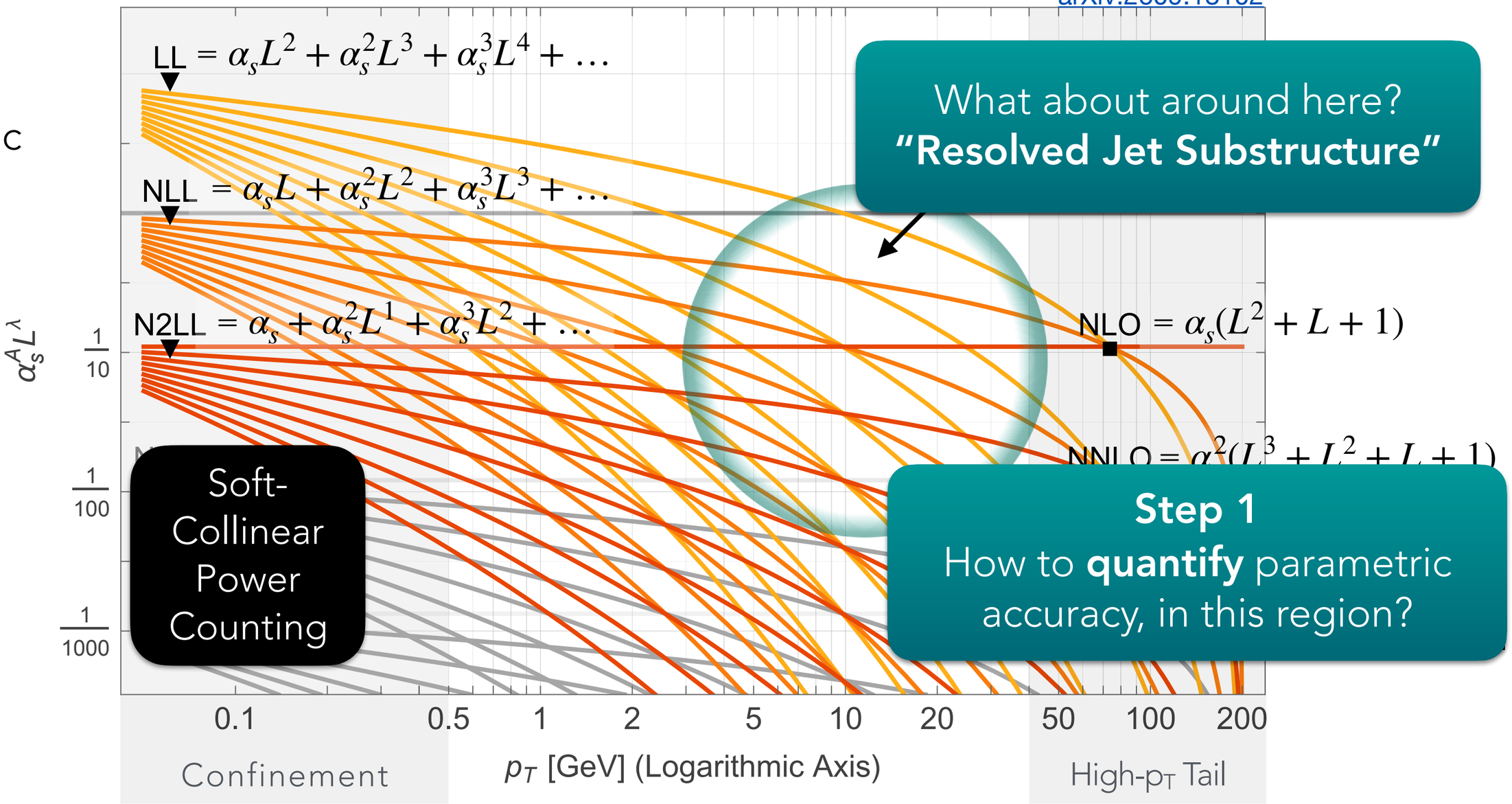


"Cat's-Cradle Diagram" of Perturbative Coefficients

Hard Scale $Q_H = 200\,\text{GeV}$, $\alpha_s(Q_H) = 0.11$, $L \equiv \ln(Q_H/p_T)$

arXiv:2609.18102

Plot each generic term $\alpha_s^A L^\lambda$



What about around here?
"Resolved Jet Substructure"

Soft-Collinear Power Counting

Step 1
How to **quantify** parametric accuracy, in this region?

⇒ **Parametric Accuracy** for a p_T-like substructure observable

Measure of relative accuracy: $R_{\text{acc}}(Q_S) = \dfrac{\text{Largest Uncontrolled Term}\,(Q_S)}{\text{Largest Controlled Term}\,(Q_S)}$

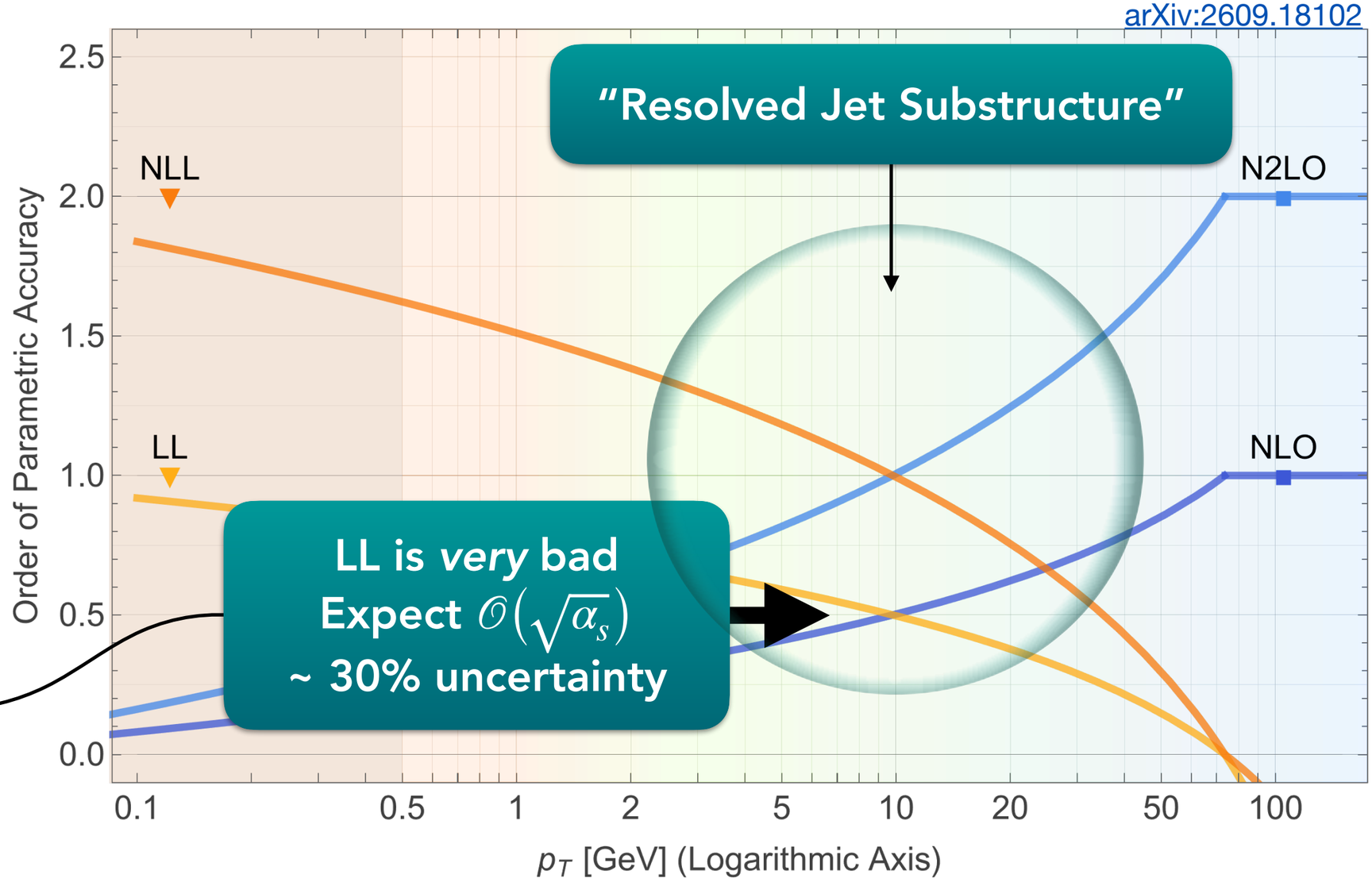
Define:

"Order of Parametric Accuracy" = equivalent number of powers of $\alpha_s \equiv \alpha_s(Q_H)$:

$$n_{\text{eff}}(Q_S) = \log_\alpha\left(R_{\text{acc}}\right)$$

But on the plots earlier, we saw order 5% **?**

arXiv:2609.18102



Resummations & Parton Showers

Resummation: anchored in phase-space region where $\alpha_s L \sim 1$

LL: $\alpha_s L^2$ in exponent + running-coupling effects (at least one loop)

NLL: includes all $\alpha_s^n L^n$ terms in the exponent as well.

Coherent Parton Showers

$\alpha_s L^2 + \alpha_s L$ in exponent + running coupling

Correct $\alpha_s^1 L^1$

+ Strict unitarity: $\int$Reals = −Virtuals

+ Exact (E,p) conservation { with ***local*** recoils ← with ***global*** recoils

Local ➜ undesired effects at $\alpha_s^2 L^2$
(But at least not ***violating*** momentum conservation)

+ Typically "CMW" scheme

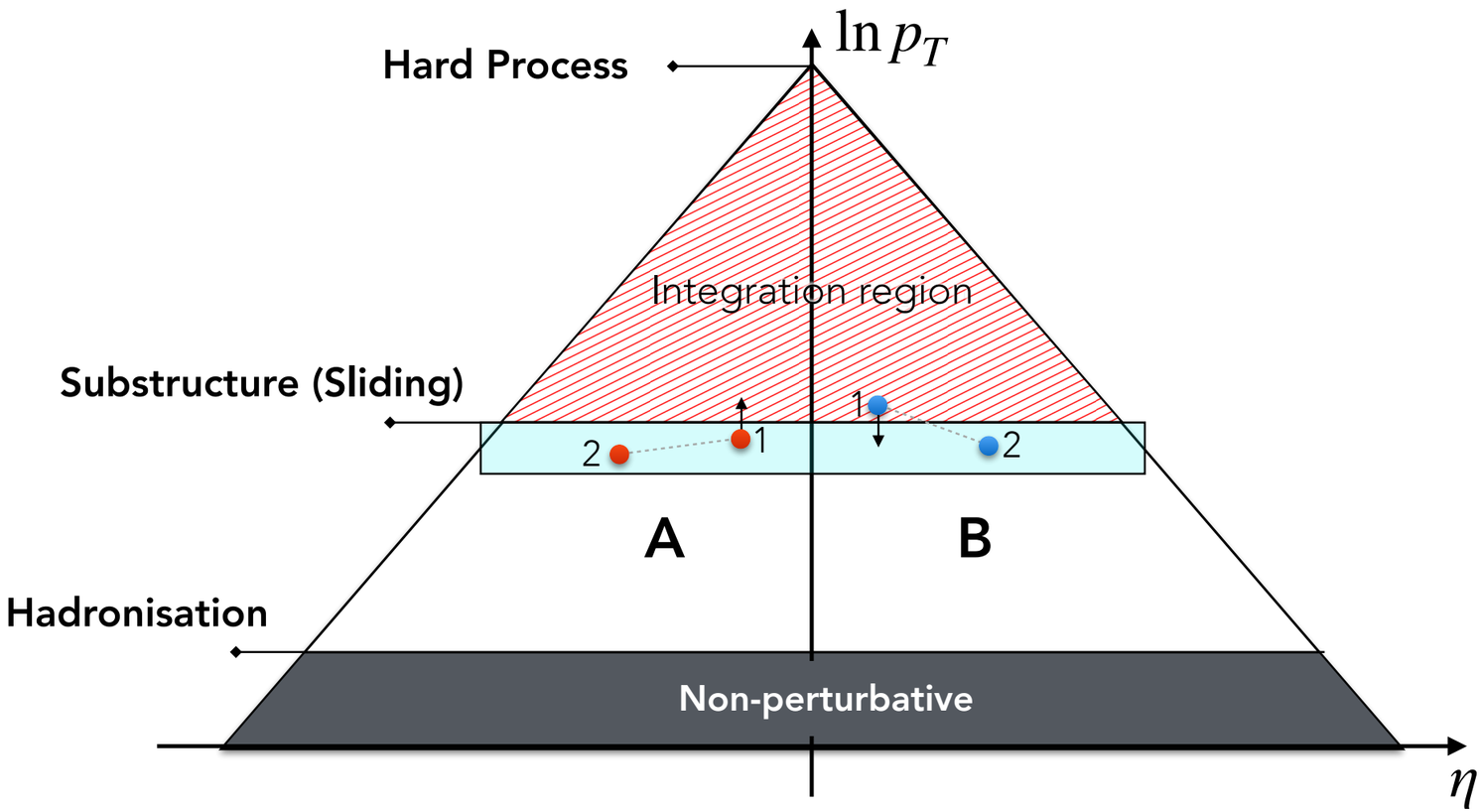
(+ Matching to high-freq. end: MECs / Matching / Merging)

(In principle, also: + "tuning", but that is not the topic here)

➜ Differences for parametric accuracies in resolved substructure region?

Momentum Conservation & Recoil Effects

The largest problematic term in local $p_\perp$-ordered dipole showers: $\alpha_s^2 L^2$



Case A: a branching (1) that was previously *not* inside the integration region, can get kicked into it by a second branching (2).

Case B: a branching (1) that was previously *inside* the integration region, can get kicked out of it by a second branching (2).

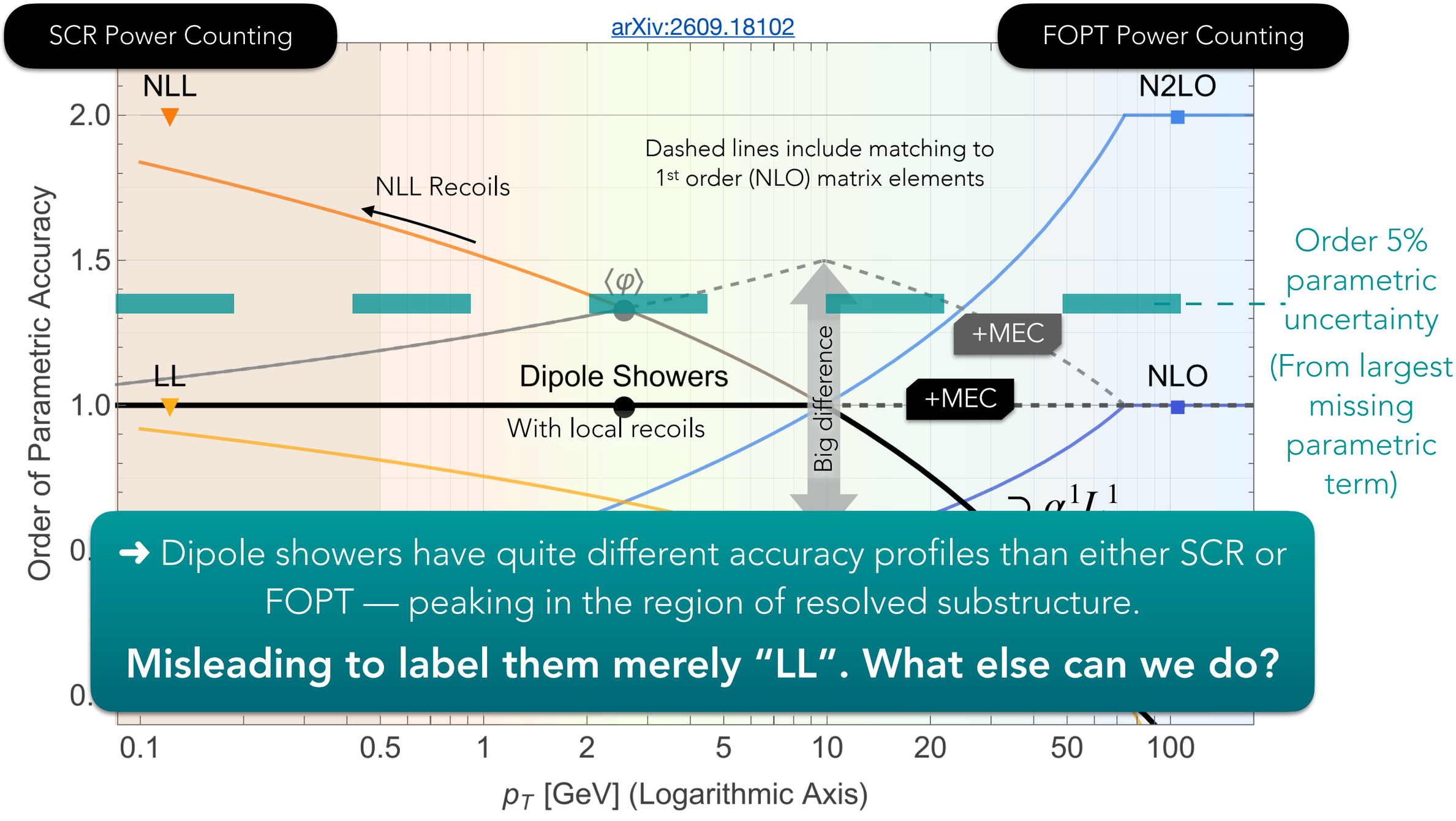
To change the integral, both branchings 1 and 2 must be near the lower boundary of the integration region ⇒ no explicit logs of p_T. But each branching can be anywhere in rapidity ⇒ $\alpha_s^2 L^2$

Remark 1: for 1-prong substructure, 2nd branching must be in $\Delta y = L/2$

Remark 2: random walk in p_T: partial compensation: Prob(A) = 2/3, Prob(B) = 1/3

⇒ Only expect ~ $\alpha_s^2 L^2/6$ for ϕ-integrated observables

1-Prong parametric accuracy of $p_\perp$-ordered dipole showers



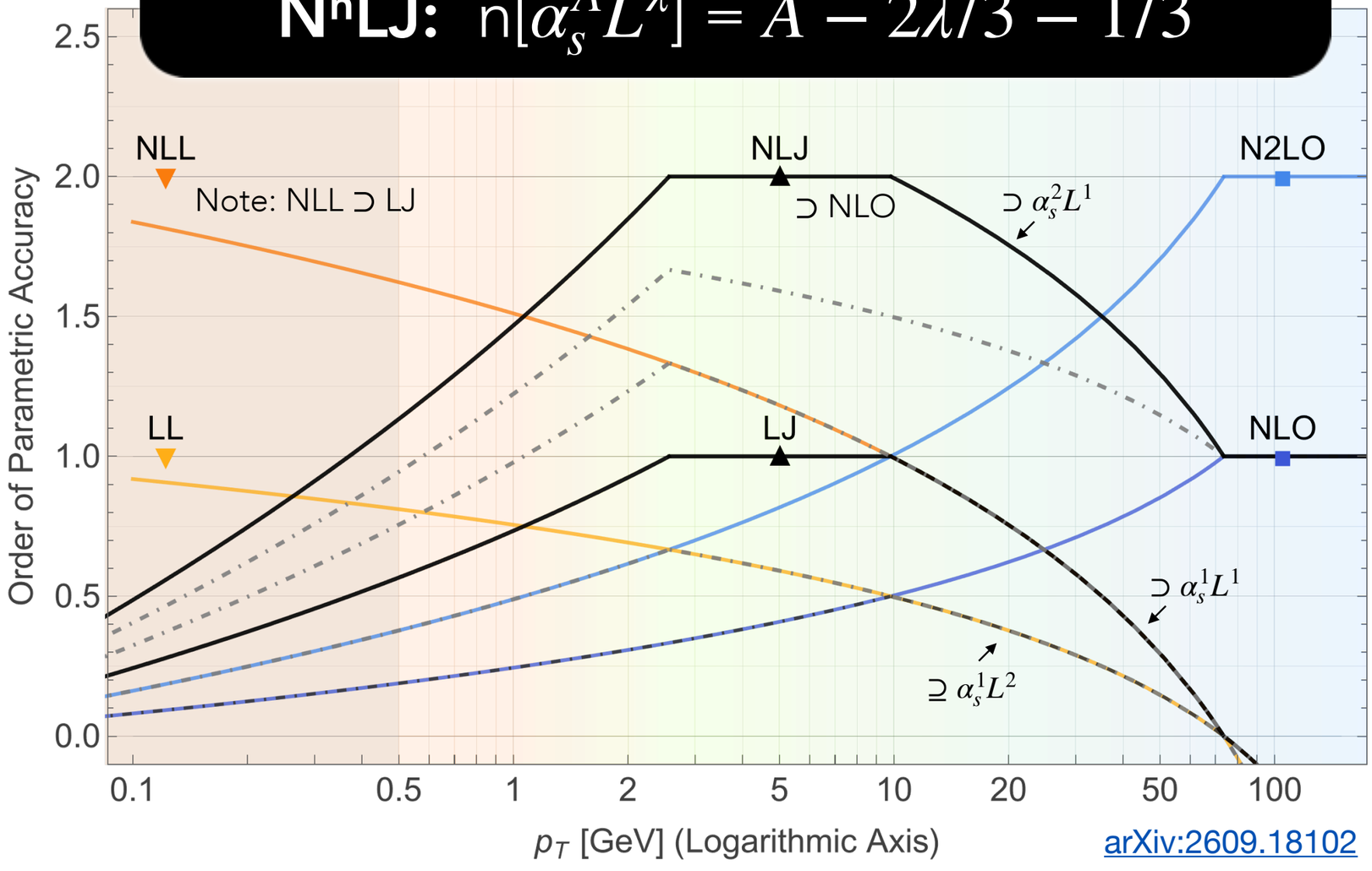
A Power Counting for Resolved Jet Substructure

Reasonable for any semi-resolved counting?

- Exponentiation of all singular terms through a fixed order in α_s
- All nonsingular terms at the preceding order
- Order-by-order progression in region of resolved substructure
- (+optionally $\mathrm{N}^{n-1}\mathrm{LL}$ asymptotics?)

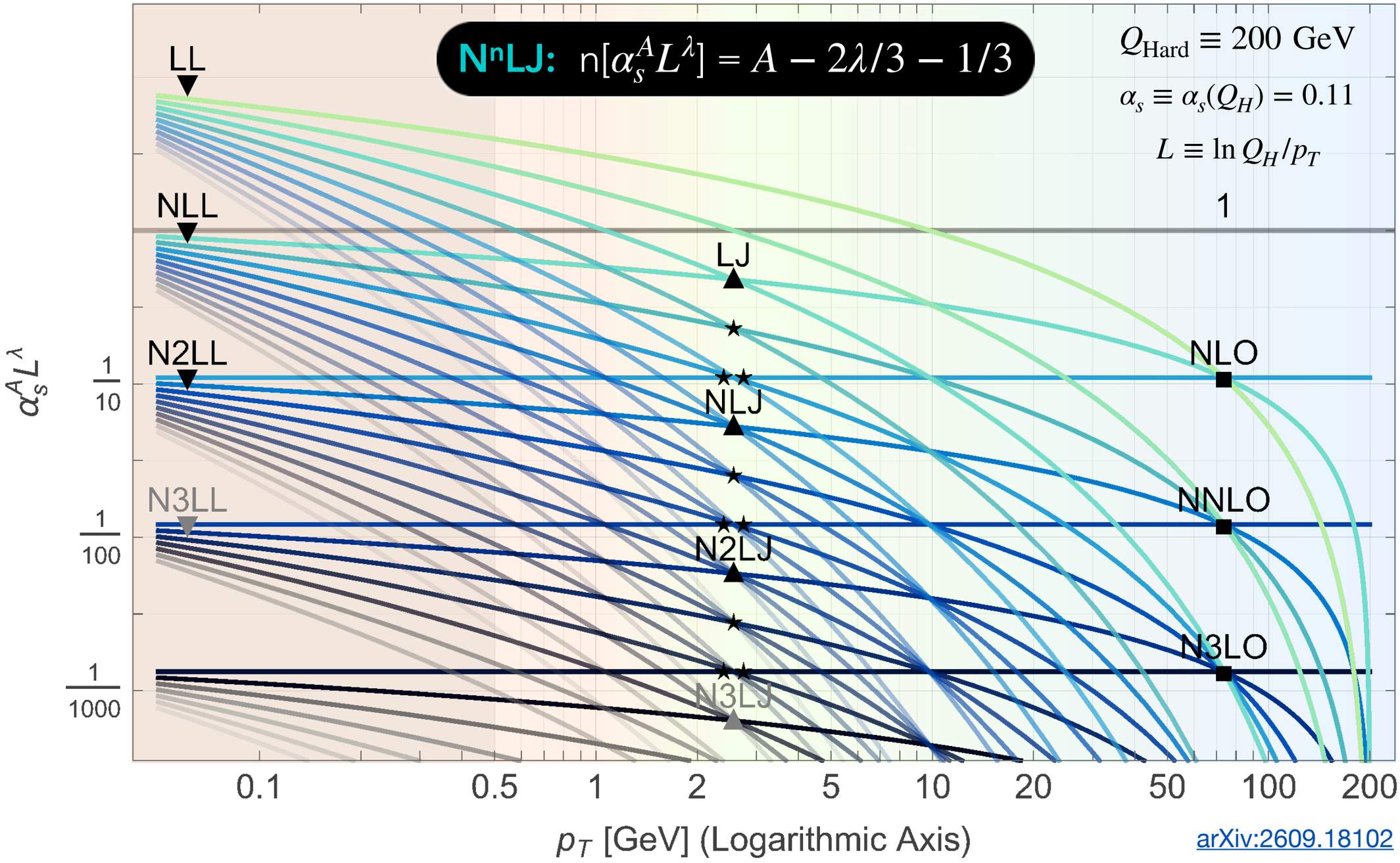
Example: **$J_{2/3}$ Order**

NⁿLJ: $\mathrm{n}[\alpha_s^A L^\lambda] = A - 2\lambda/3 - 1/3$



arXiv:2609.18102

Cat's-Cradle with $J_{3/2}$ coloring



arXiv:2609.18102

Sets out a research program I think is important

For resolved jet substructure,

It makes sense to target each NLL coefficient one after another

DLA	$\alpha_s L^2$			
DLA⋆	$+\ \alpha_s^2 L^3$			
LJ	$+\ \alpha_s^3 L^4,$	$\alpha_s L,$		
LJ⋆	$+\ \alpha_s^4 L^5,$	$\alpha_s^2 L^2$		$2 \to 4$ Branchers
LJ⋆⋆	$+\ \alpha_s^5 L^6,$	$\alpha_s^3 L^3,$	$\alpha_s L^0$	$2 \to 5$ Branchers (& NLO Matching)
NLJ	$+\ \alpha_s^6 L^7,$	$\alpha_s^4 L^4,$	$\alpha_s^2 L^1$	"Jet Recoils" & NNLO Matching
NLJ⋆	$+\ \alpha_s^7 L^8,$	$\alpha_s^5 L^5,$	$\alpha_s^3 L^2$	
NLJ⋆⋆	$+\ \alpha_s^8 L^9,$	$\alpha_s^6 L^6,$	$\alpha_s^4 L^3,$	$\alpha_s^2 L^0$
NNLJ	$+\ \alpha_s^9 L^{10},$	$\alpha_s^7 L^7,$	$\alpha_s^5 L^4,$	$\alpha_s^3 L^1$

And then each NNLL one

Current Projects

(+ anything that is parametrically of similar order, including $1/N_C^2$, power corrections, …)

Percent-level target for LHC p_T-substructure predictions

More general observables & Two-prong substructure

(Obviously, also need to go to non-exponentiated level, and broader observables)

Current Work

❶ NNLO ⊕ QCD/QED Antenna Showers for $Z^0 \to$ Hadrons

First fully-differential NNLO + shower (details could fill a talk on their own)

Bremsstrahlung poles of shower algorithm **fully matched to NNLO singularity structure**

(MiNNLOPS & Geneva employ showers with NLO pole structures + inclusive inputs)

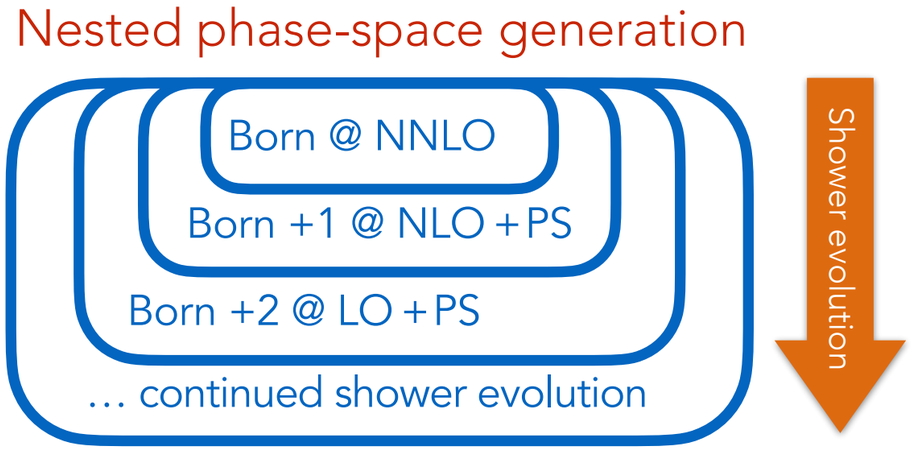
Long time coming

Iterated tree-level ME corrections 1102.2126

One-loop $2 \to 3$ ME corrections 1303.4974

Direct $2 \to 4$ branchings 1611.00013

2-Loop Born & Double-Real Sectorisation 2108.07133 2412.14242



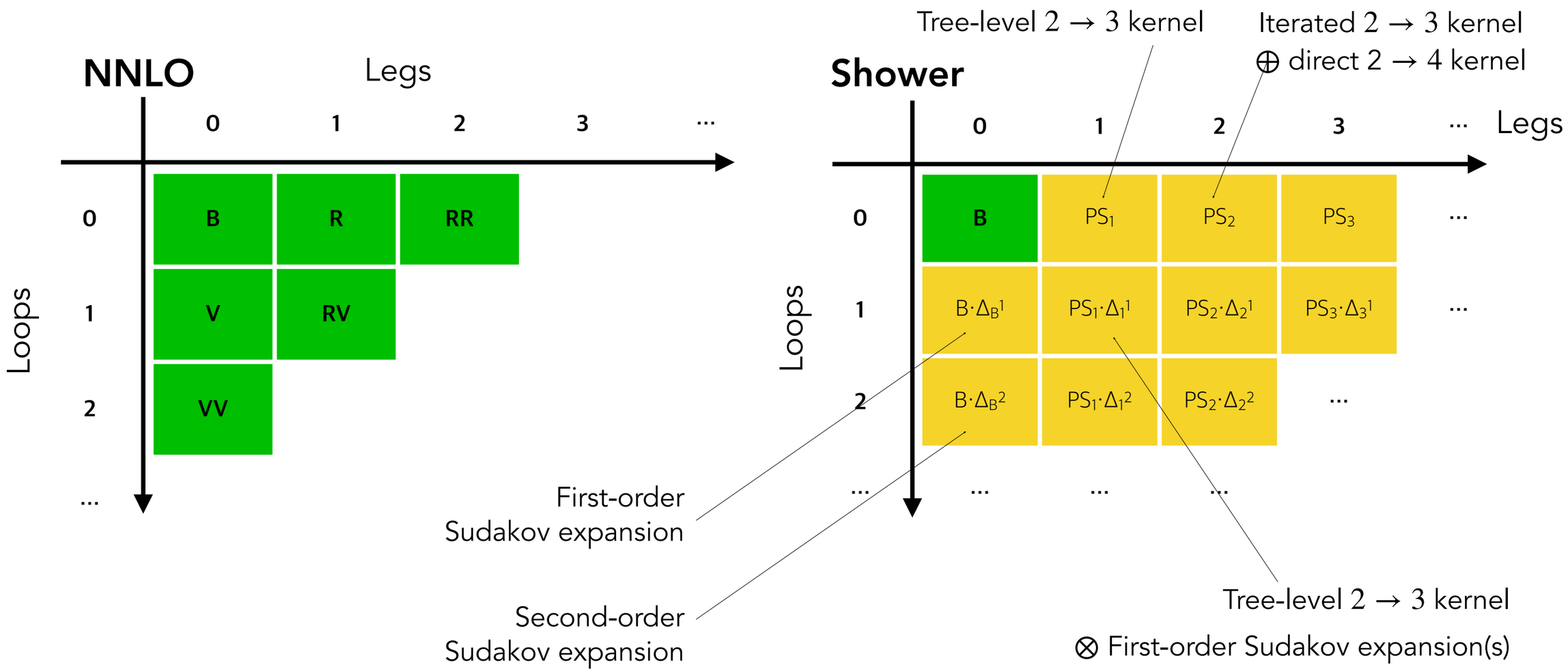
Should be able to reach NNLO + NLJ + NLL accuracy with "Jet Recoils 2512.07370

➜ new standard for constraining **non-perturbative (string-fragmentation) models**

Important for non-perturbative studies & non-perturbative uncertainties at LHC

Coefficients of the Perturbative Expansions

Shower "observable" ≡ fully differential jet rates in each of the (nested) phase spaces

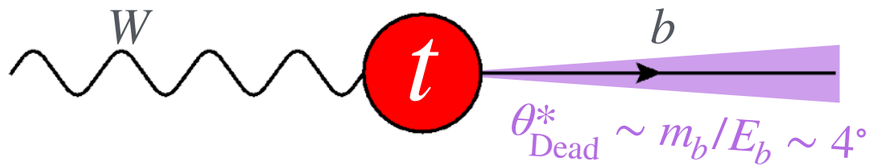


Note: shower coefficients **not independent** — tied together by universality (→) and unitarity (↙)!

❷ Bremsstrahlung in Top-Quark Decays

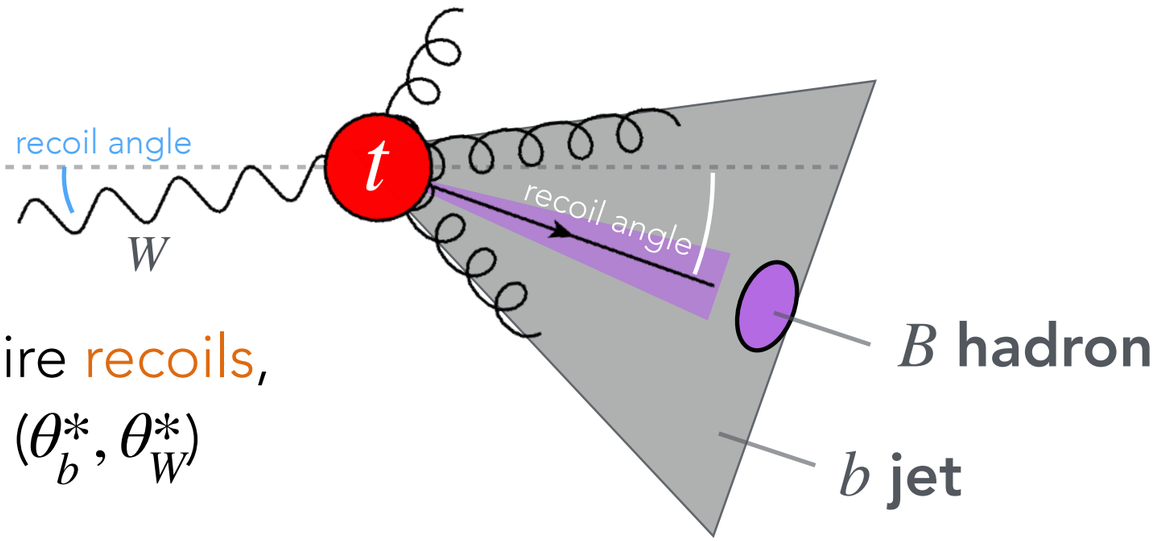
Born level $t \to Wb$, in top rest frame

$E_W \sim 105\,\text{GeV}$,
$E_b \sim 67\,\text{GeV}$



$$m_t^2 = m_W^2 + m_b^2 + 2(p_W \cdot p_b)$$

⊕ Radiative corrections



(E,p) conservation $\Longrightarrow$ W & b acquire recoils,
in both energy (x_b^*, x_W^*) and angle (θ_b^*, θ_W^*)

↳ Reconstruct m_t

Current State of the Art at LHC

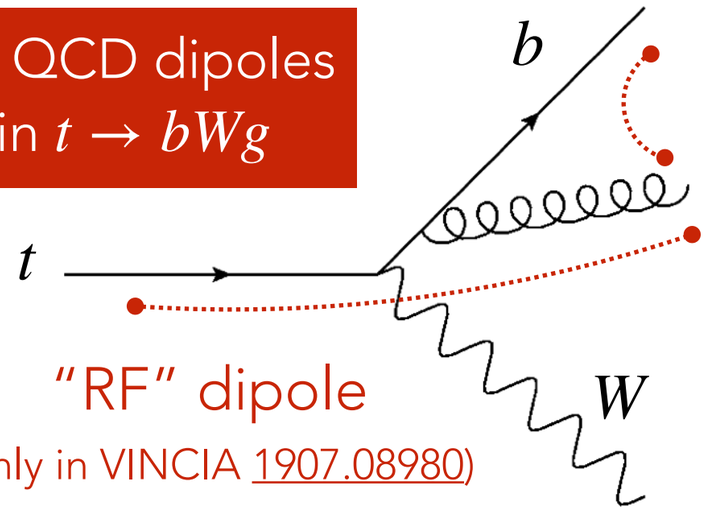
1st (hardest) emission corrected by tree-level matrix element (e.g., POWHEG)

2nd and subsequent emissions: parton shower

Substantial theory uncertainties on reconstructed m_t from ambiguities on **2nd emission**

❷ 2nd-Order ME Corrections in Top-Quark Decays

LC QCD dipoles in $t \to bWg$



"RF" dipole

(Only in VINCIA 1907.08980)

Gabriel Love 2026 (Monash PhD student)**:**

First direct $2 \to 4$ for RF

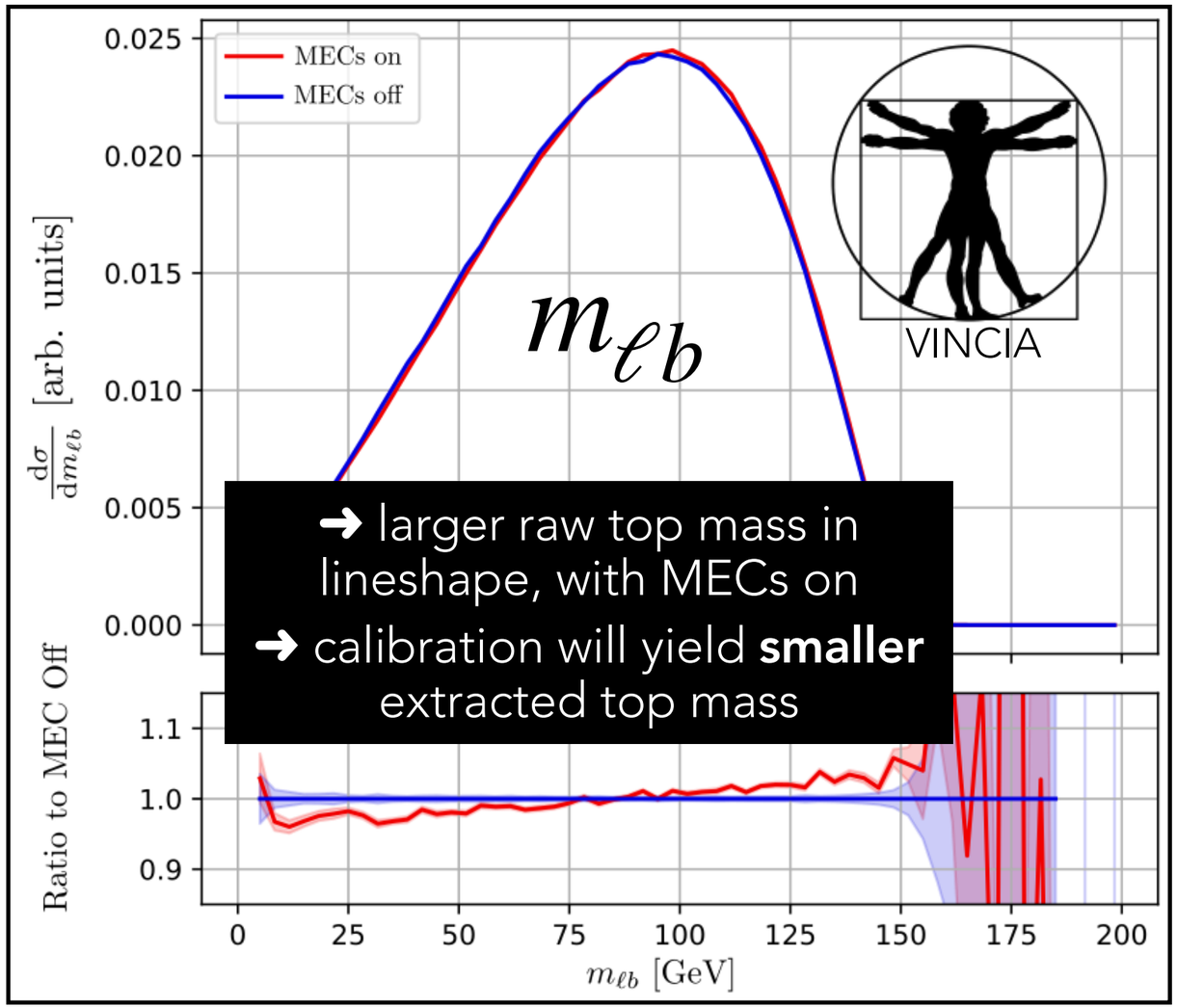
+ ME-corrected branchings for 1st **and 2nd** branchings

Validated at 2nd order with Sunshine [2507.00111]

Paper should be out soon!

The **VINCIA** antenna shower has improved coherence & recoils through its "RF" antennae

But previously did not have ME Corrections



➜ larger raw top mass in lineshape, with MECs on

➜ calibration will yield **smaller** extracted top mass

Conclusions

Physics of (QCD) Jet Substructure = Physics of Bremsstrahlung

⊕ Matching to hard process (POW!) for high-scale substructure / extra hard jets

Billions of jets in LHC data ➜ small stat. uncertainties

Limited theoretical accuracy ➜ important systematics

So far, two main foci for theory improvements in MC models:

- **Past decades:** Matching to hard process (FOPT)
- **More recently:** improving soft-collinear accuracy (SCR)

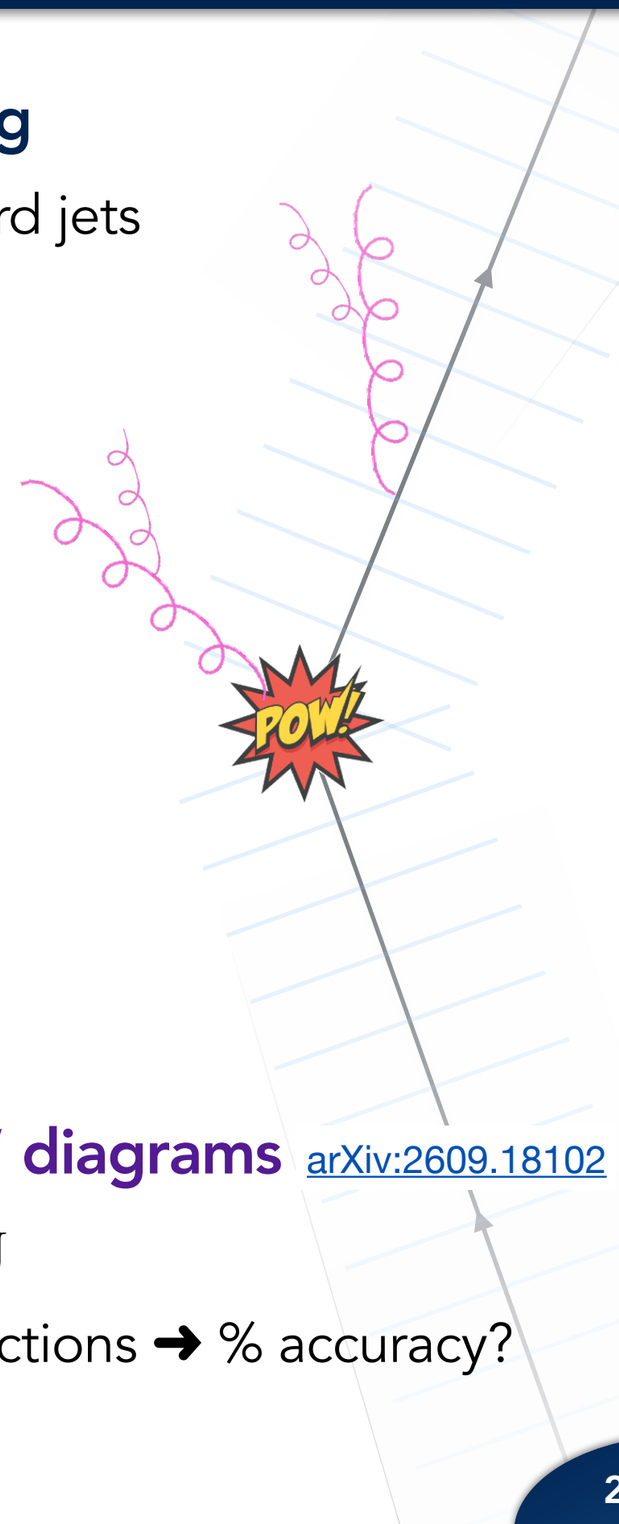
I want to call attention to the resolved-substructure region

- ≈ midway between the two (FOPT & SCR) extremes

Jumble of perturbative coefficients analysed via "cats-cradle" diagrams arXiv:2609.18102

➜ Design power countings targeting this region specifically, e.g., $\mathrm{N^nLJ}$

⇒ Program of NNLO + NLJ (via 2nd-order shower kernels) + NLL corrections ➜ % accuracy?



And colors are pretty, too



Extra Slides

Outlook

Perturbative accuracy measures for resolved kinematics

Reconciles empirical observation that dipole showers tend to do significantly better than pure LL for resolved kinematics: they are LJ + LL; not just LL.

See also Höche, Reichelt, Siegert JHEP 01 (2018) 118

Emphasises a further desirable property

Having a large region of ~ stable parametric accuracy for resolved observables

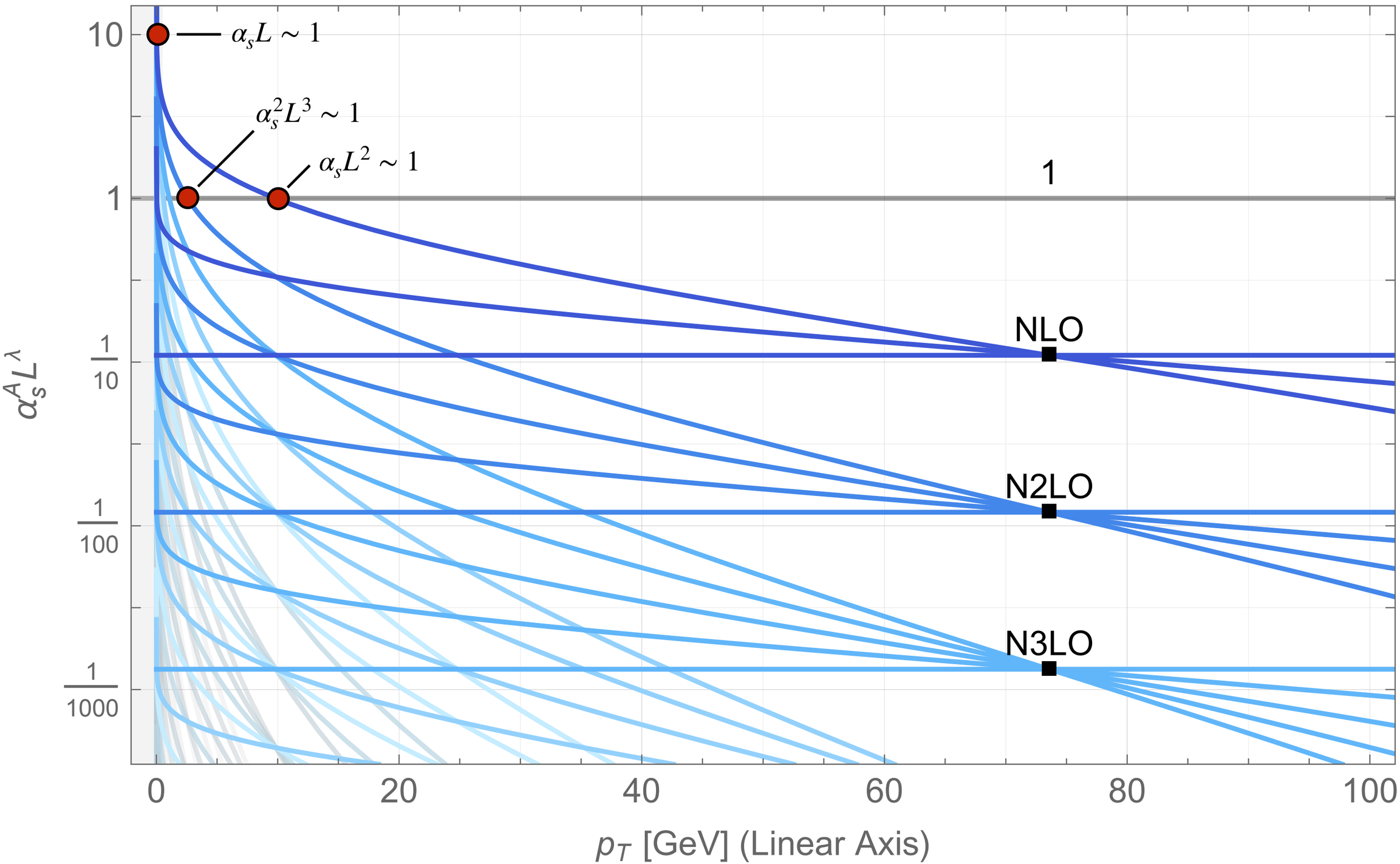
Outlook

Go to cross-section level, more general & multi-prong observables.

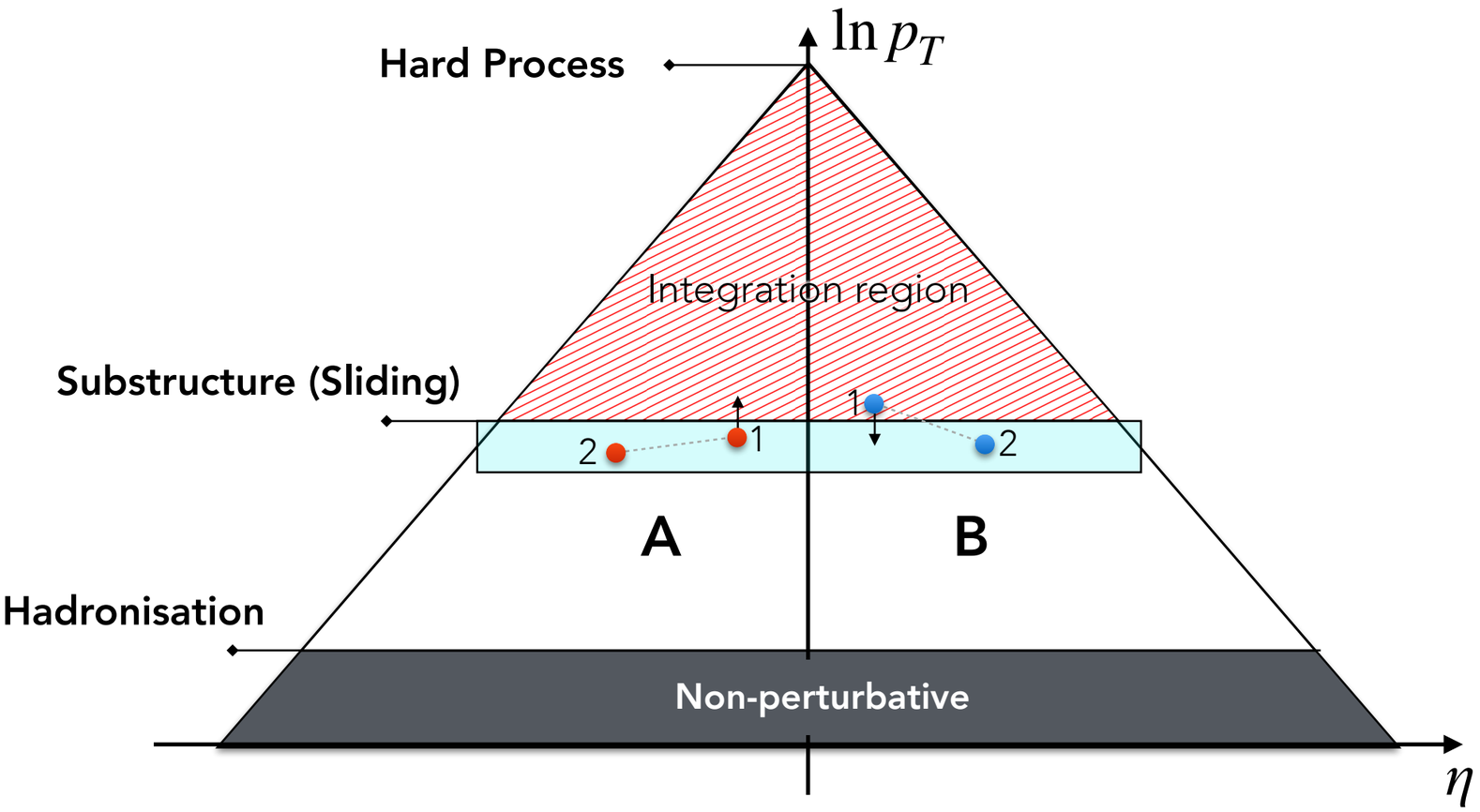
Scrutinise O(1) coefficient assumption beyond NLO, absorb α_s running effects

Power corrections, subleading N_C, …

Parametric accuracy ↔ uncertainty estimates


$\alpha_s L \sim 1$
$\alpha_s^2 L^3 \sim 1$
$\alpha_s L^2 \sim 1$
1
NLO
N2LO
N3LO
10
1
$\frac{1}{10}$
$\frac{1}{100}$
$\frac{1}{1000}$
$\alpha_s^A L^\lambda$
0
20
40
60
80
100
p_T [GeV] (Linear Axis)

Momentum Conservation & Recoil Effects



Case A: a branching (1) that was previously *not* inside the integration region, can get kicked into it by a second branching (2).

Case B: a branching (1) that was previously *inside* the integration region, can get kicked out of it by a second branching (2).

To change the integral, both branchings 1 and 2 must be near the lower boundary of the integration region ⇒ no explicit logs of p_T. But each branching can be anywhere in rapidity ⇒ $\alpha_s^2 L^2$