Computing Massive 1→3 Splitting Kernels for Future Parton Shower Models

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Motivation

As the field moves towards higher precision searches and measurements, sub-leading corrections become important.

We employ new techniques in deriving splitting kernels, treating three corrections: massive quark effects, triple-collinear effects, and soft/collinear phase-space overlap.

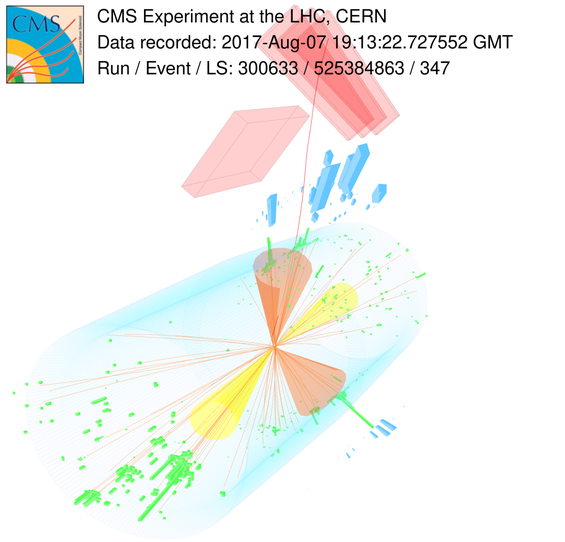

Figure 1. CMS measurement of an HH→4b event [1]. Massive effects are relevant in b-jets, and such events are important in measurements of Higgs self-coupling.

Triple-collinear splittings are needed for a full NNLL treatment in Monte Carlo simulation. Moreover, massive effects contribute to many of the most common and important LHC searches and backgrounds, particularly b-jet events. Incorporating mass at the shower level may also contribute to more accurate models for hadronization.

DGLAP Splitting Kernels

- Splitting kernels describe the probability of different splits occurring at each step in the parton shower
- This is the tool which Monte Carlo generators use to build the parton shower
- Well-known forms for 1→2 splittings can be calculated using limit-dependent methods [4], with corrections for massive quarks [5]. The $1 \to 3$ splittings can be found similarly.

$$\langle \hat{P}_{qg}(z,\epsilon) \rangle = C_F \left[\frac{1+z^2}{1-z} - \epsilon(1-z) - 2\frac{m_Q^2}{\tilde{s}_{12}} \right]$$

$$\langle \hat{P}_{q\bar{q}}(z,\epsilon) \rangle = T_R \left[1 - \frac{2z(1-z)}{1-\epsilon} - \frac{m_Q^2}{s_{12}} \right]$$

$$\langle \hat{P}_{gg}(z,\epsilon) \rangle = 2C_A \left[\frac{z}{1-z} + \frac{1-z}{z} + z(1-z) \right]$$

- Higher order calculations require the $1 \to 3$ splitting kernels, with kinematics in the triple-collinear limit
- Additional corrections from loop diagrams and loop emissions can also be considered
- The massless $1 \to 3$ forms can instead be derived from the Berends and Giele recursions, while identifying which components contribute to the soft/collinear overlap in phase space [3]

Scalar and Fermionic Vertex Components

The structure of QCD interactions can inherently be split into parts, isolating the underlying scalar and fermionic structures [3]. In particular, the scalar interaction and decay vertices

$$S^\mu(p_i, p_j) = \frac{(2p_i + p_j)^\mu}{p_{ij}^2}$$

$$D^\mu(p_i, p_j, n) = d^\mu_\nu(p_{ij}, n)\frac{(p_i - p_j)^\nu}{p_{ij}^2},$$

reminiscent of the forms from Scalar QED; and the fermionic/magnetic interaction and decay vertices

$$M^\mu(p_i, p_j) = \frac{i\sigma^{\mu\nu}}{p_{ij}^2} p_{j\nu}$$

$$F^\mu(p_i, p_j, n) = d^{\mu\nu}(p_{ij}, n)\frac{\gamma_\nu}{p_{ij}^2}.$$

The Berends and Giele recursion relations [2] can be rewritten in terms of these structures, and then used to calculate splitting kernels [3].

This calculation **doesn't rely on kinematical limits**, but they can be imposed, and return the well-known forms.

All of the soft/collinear overlap in phase space is contained within the purely scalar terms. This enables easy subtraction of double-countings from each limit.

Berends and Giele Recursive Currents

QCD interactions can be modeled by taking n-particle currents, and splitting them recursively. This follows a similar logic to that of splitting functions.

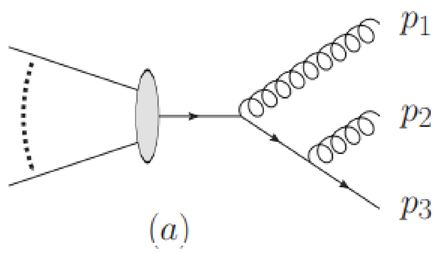

Figure 2. Example of a $1 \to 3$ splitting diagram, in terms of final state momenta [3]. The set of momenta can be partitioned into currents at each step.

The currents are defined by partitioning sets of incoming momenta into subsets of outgoing momenta.

- The current for a single gluon is the **polarization tensor** $J_\xi(p) = \epsilon_\xi(p)$
- The current for a single quark is the **spinor** $J_s(Q) = u_s(Q)$

For a two-particle final state, the recursive result follows the QCD vertex structure, adding a propagator for the incoming particle.

$$J^x_\xi(p_1, p_2) = \frac{igf^{abx}}{(p_1+p_2)^2}\Gamma^{\mu,\nu}_x(p_1, p_2, -(p_1+p_2))J_\mu(p_1)J_\nu(p_2)$$

$$J(Q,p) = iT^a_{ij}u(Q)\not{\epsilon}(p)\frac{i}{\not{Q}+\not{p}-m} = -J(Q)\not{J}(p)\frac{1}{\not{Q}+\not{p}-m}.$$

From these, and terms for the $g \to qq$ splitting and the four-gluon vertex, the colour-dressed forms for n-gluon final states can be written recursively [2].

These currents can then be rewritten in order to separate the underlying structures of scalar radiators and decays, and fermionic interactions [3].

Splitting Kernels From Recursion

Working from the Berends & Giele recursion relations, the splitting functions can be taken as [3]

$$P_q^{ss'}(1,...n) = \delta^{ss'}\left(\frac{s_{1...n} - m^2_{1...n}}{8\pi\mu_0^{2\epsilon}\alpha_S}\right)^{n-1}\frac{\mathsf{Tr}\left[\not{p}\Psi(\{p_1,...,p_n\})\overline{\Psi}(\{p_1,...,p_n\})\right]}{\mathsf{Tr}\left[\not{p}\Psi(p_{1...n}]\,\overline{\Psi}(p_{1...n})\right.}$$

$$P_g^{\mu\nu}(1,...,n) = \frac{D-2}{2}\left(\frac{s_{1...n} - m^2_{1...n}}{8\pi\mu_0^{2\epsilon}\alpha_S}\right)^{n-1}\frac{d^{\mu\rho}(p_{1...n},n)J_\rho(\{p_1,...,p_n\})J^\dagger_\sigma(\{p_1,...,p_n\})d^{\sigma\nu}(p_{1...n},n)}{d^{\kappa\lambda}(\overline{p}_{1...n},n)J_\lambda(\overline{p}_{1...n})J^\dagger_\tau(\overline{p}_{1...n})D^\tau_\kappa(\overline{p}_{1...n},n)}.$$

For the $q \to gq$ splitting, for example, this yields a clear set of scalar and interaction terms [3], which give the well-known forms of the splitting kernel under the (massless) collinear limit $p_1^2, p_2^2 \to 0$

$$P_q(p_1,p_2) = C_F\frac{2z_1}{z_2}\left(1 - \frac{p_1^2}{p_{12}^2}\frac{z_{12}}{z_1} - \frac{p_2^2}{p_{12}^2}\frac{z_{12}}{z_2}\right) + C_F(1-\epsilon)\left(\frac{z_2}{z_{12}} - \frac{z_2}{z_1}\frac{p_1^2}{p_{12}^2} - \frac{p_2^2}{p_{12}^2}\right).$$

The massive $q \to gq$ splitting kernel can be found by the same method as the massless derivation above. We implement a different parametrization of the quark momenta, so the result agrees with the well-known form under the expected limits $p_1^2, m^2 \to m_Q^2$, $p_2^2 \to 0$.

$$\mathbf{P_q(p_1,p_2)} = \mathbf{(1-\epsilon)}\left(\mathbf{z_2} - \frac{\mathbf{1}}{\tilde{\mathbf{s}}_{\mathbf{12}}}\left(\mathbf{m^2 - p_1^2 + p_2^2}\right)\right) + \frac{\mathbf{1}}{\tilde{\mathbf{s}}_{\mathbf{12}}}\frac{\mathbf{2z_1}}{\mathbf{z_2}}\left(-\frac{\mathbf{z_2}}{\mathbf{z_1}}\mathbf{p_1^2} - \frac{\mathbf{z_1}}{\mathbf{z_2}}\mathbf{p_2^2} + \tilde{\mathbf{s}}_{\mathbf{12}}\right).$$

This method of deriving the splitting kernels provides greater precision over current $1 \to 3$ massive splitting models by including subleading terms. Full implementation will account for massive, triple-collinear, and overlapping soft/collinear effects.

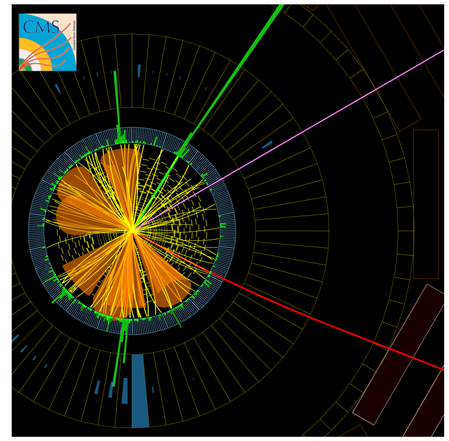

Figure 3. CMS candidate of a di-lepton ttH event [6]. This interaction is important for measuring the Yukawa coupling of the Higgs, and relies on top-initiated massive b-quark showers.

Massive quark-initiated jets are significant in collider physics, both as search targets and backgrounds. Implementing models of massive splittings into Monte Carlo event generators will improve accuracy and precision in simulating these events.

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