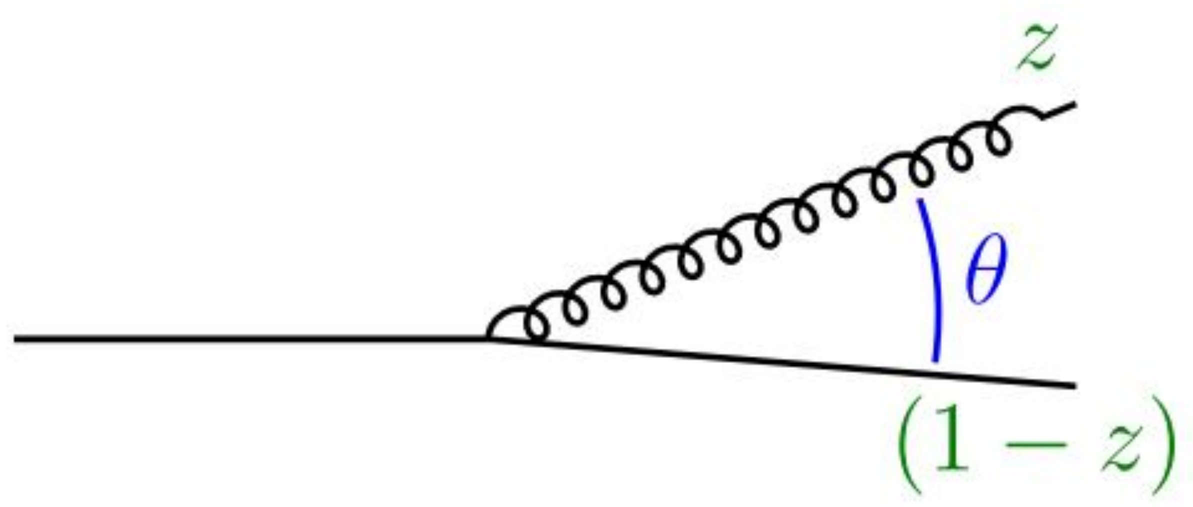
The Lund b-jet plane

BOOST 2025 @ Brown University, Providence.
In collaboration with S. Marzani and G. Soyez

Andrea Ghira, Università di Genova and INFN, 29/07/2025

Soft and collinear limit in massless QCD

At high energies, soft ($z \to 0$) and collinear ($\theta^2 \to 0$) emissions are favored in QCD



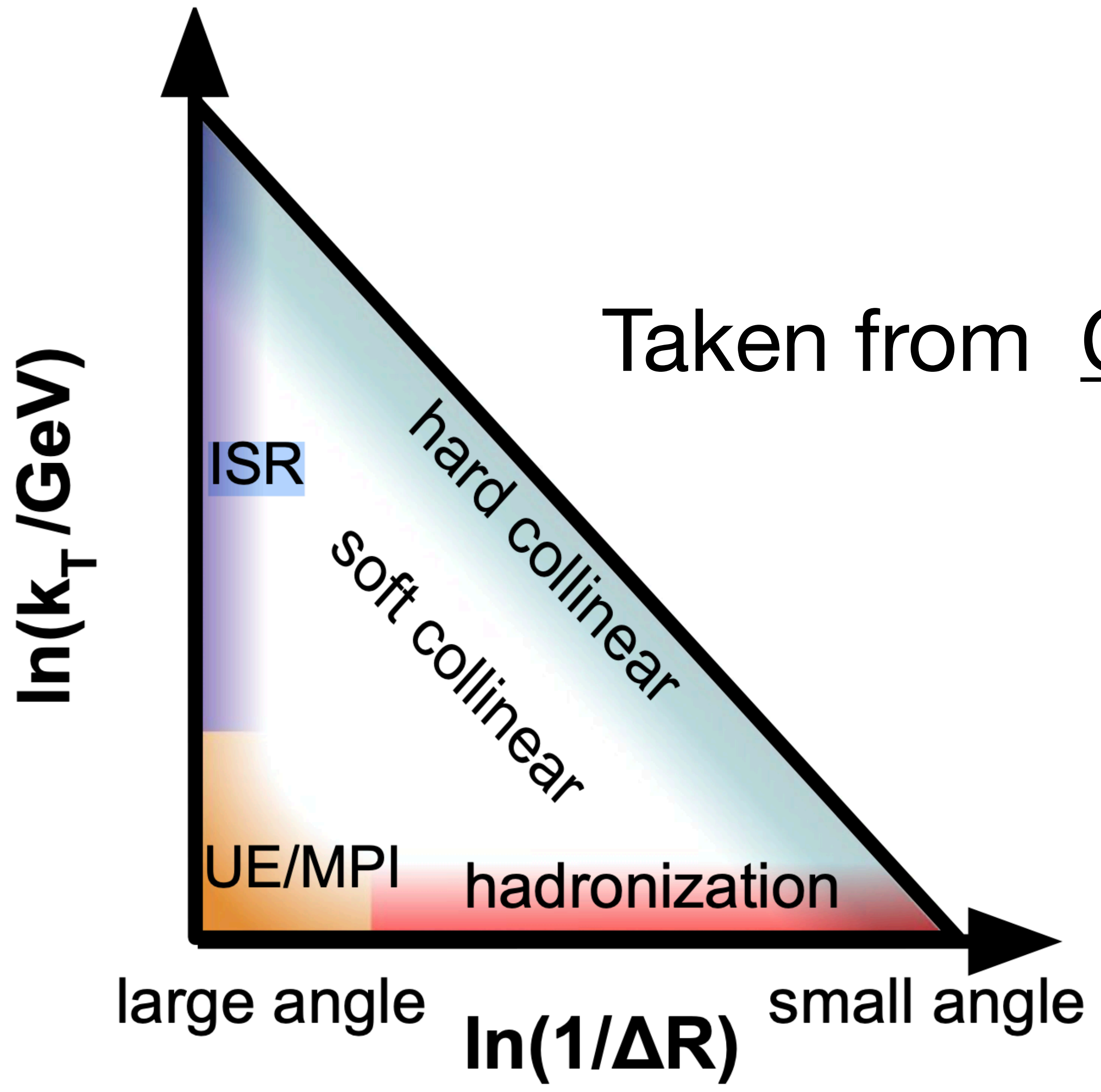
$$d\sigma \simeq \frac{\alpha_S C_F}{2\pi}\frac{dk_t^2}{k_t^2}\frac{d\theta^2}{\theta^2} = \frac{2\alpha_S C_F}{\pi} d\log k_t \, d\log\frac{1}{\theta}$$

z= gluon energy fraction

θ= gluon splitting angle

k_t = gluon transverse momentum

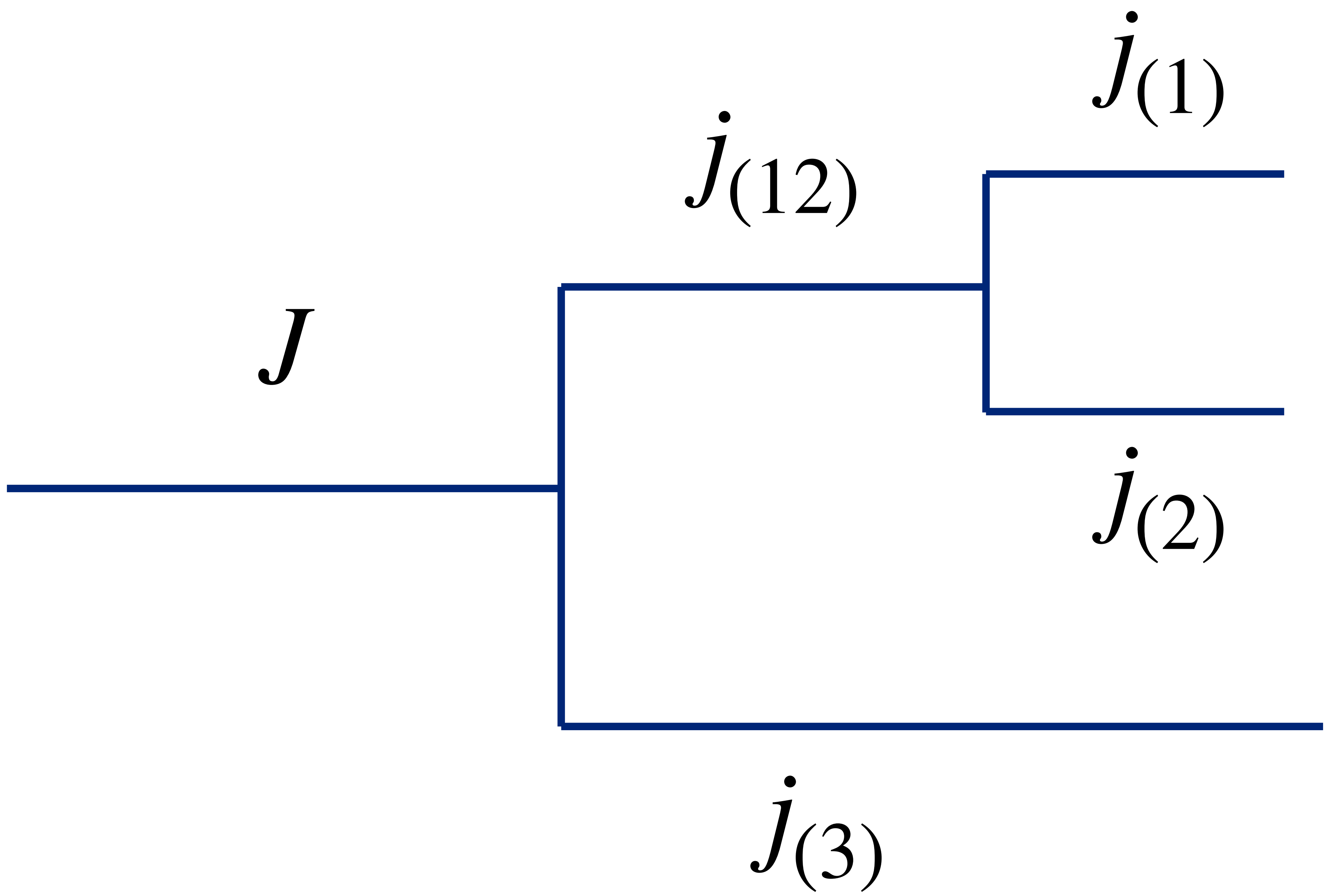
Soft and collinear emissions are uniformly distributed in a log-log plane →Lund Plane



Taken from CMS

Lund plane as an observable

We assume to have an anti-k_t jet with an intrinsic hard scale $Q\ (p_\perp, E_J, \ldots)$, reclustered with C/A.



1. Undo the last step of the declustering. Assume $j_{(12)}$ to be the hardest branch.

2. Record the properties of the branching:

- Δ = angular separation,

- k_t = transverse momentum of $j_{(3)}$ wrt $j_{(12)}$,

- z = momentum fraction of $j_{(3)}$ wrt $j_{(12)}$

3. Iterate following the hardest branch

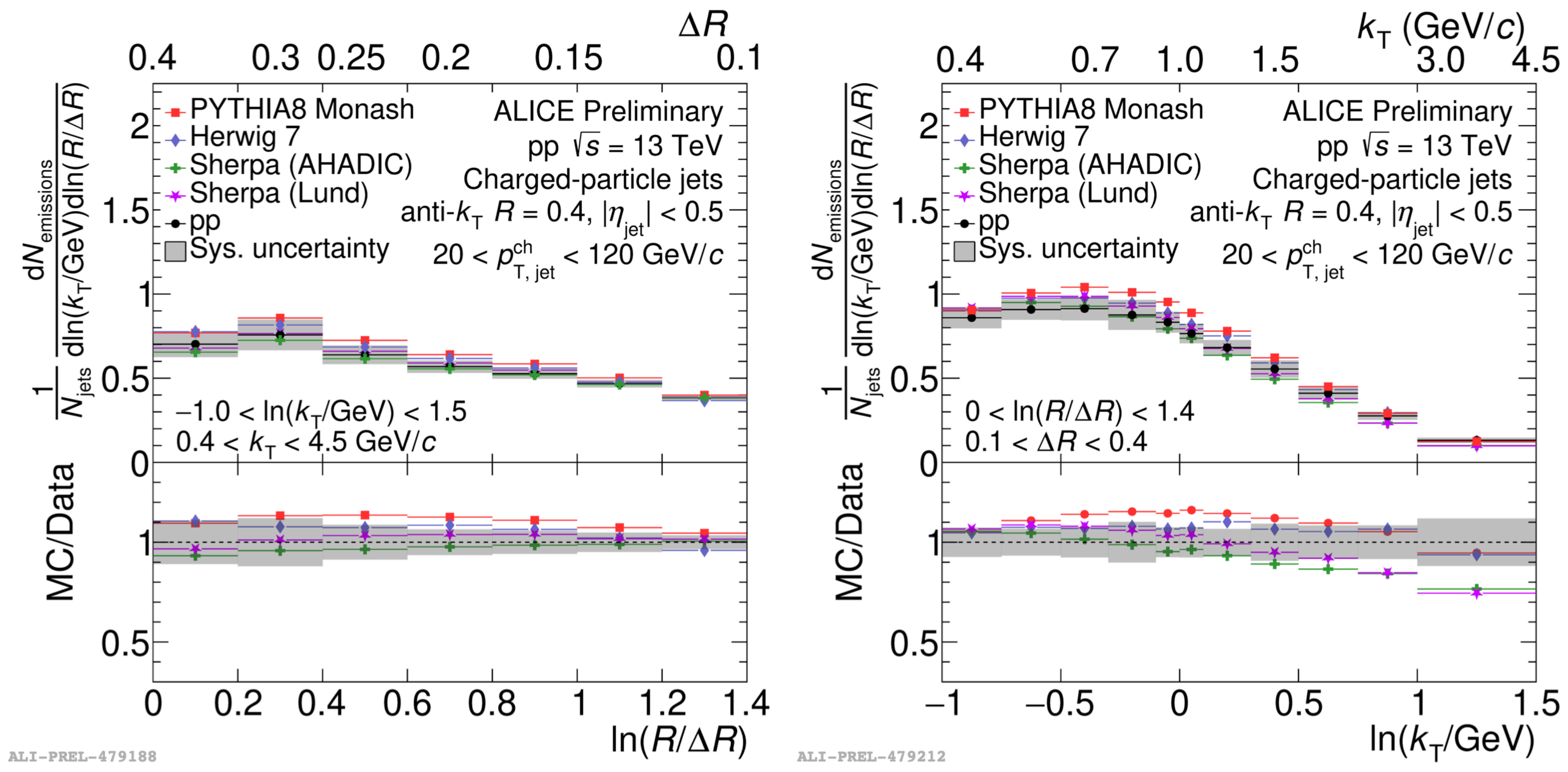
Primary Lund Plane density

The primary Lund plane density is defined as

$$\rho = \frac{1}{N_J} \frac{d^2 n}{d\log\left(\frac{1}{\Delta}\right) d\log k_t}$$

n = number of emissions

N_J = number of jets



Plots taken from ALICE

- First measurement present by ATLAS and then also by ALICE and CMS
- First calculation for massless jets performed by A. Lifson, G. Salam and G. Soyez
- Recent first measurement on heavy flavor jets done by LCHb collaboration
- Missing calculation for Heavy flavor jets

Calculation at fixed order

- We start considering the splitting process $I \to k(z) + j(1-z)$ (assume $z > 1/2$)

- When dealing with massive quarks, we employ the so called quasi-collinear limit $k_t \sim m \ll Q$: quasi collinear factorization

$$\rho_I^{\text{Q.C.}} = \frac{\alpha_S}{\pi} \sum_k \frac{z^2(1-z)^2 k_t^2}{z^2 k_t^2 + (1-z)m_k^2 + z m_j^2 - z(1-z)m_I^2} P_{kI}(z)\,|_{1-z=\frac{k_t}{Q\Delta}}$$

$$= \frac{\alpha_S}{\pi}(1-z)\mathscr{P}_I(z)\,|_{1-z=\frac{kt}{xQ\Delta}}$$

$z \to 1$

$$\rho_q^{\text{Q.C.}} = \frac{2\alpha_S C_F}{\pi} + \mathcal{O}(1-z)$$

$$\rho_{\mathcal{Q}}^{\text{Q.C.}} = \frac{2\alpha_S C_F}{\pi} \left(\frac{\Delta^2}{\Delta^2 + \xi} \right)^2 + \mathcal{O}(1-z)$$

$$\rho_g^{\text{Q.C.}} = \frac{2\alpha_S C_A}{\pi} + \mathcal{O}(1-z)$$

$\rho_{\mathcal{Q}}$ receives suppression due to dead cone effect

$$\xi = \frac{m^2}{Q^2}$$

Multiple resummations

At higher orders we have different source of large logarithms:
$\alpha_S^{n+1} \log^m \Delta \log^p k_t \log^s \xi, \quad m+p+s<n$. This logs have different physical origins:

- Running coupling corrections, enhanced by logs of k_T,

$$\rho_I^{\text{Q.C.}} \to \frac{\alpha_S^{\text{CMW}}(k_t^2)}{\pi} \sum_k \frac{z^2(1-z)^2 k_t^2}{z^2 k_t^2 + (1-z)m_k^2 + z m_j^2 - z(1-z)m_I^2} P_{kI}(z)\,|_{1-z=\frac{k_t}{Q\Delta}}$$

- Logarithms of the emission angle Δ, which induce flavor changing effects,

- Mass logs coming from virtual contribution (decoupling scheme employed in α_S) and from collinear physics,

- Soft emission at large angle and commensurate angles generates logs of $Q\Delta/k_T$.

Resummation of collinear logarithms

Hard collinear splittings can change the flavor of the hardest branch in two ways:

- $q \to qg, \mathcal{Q} \to \mathcal{Q}g$ splittings and the gluons is the hardest particle
- $g \to q\bar{q}, g \to \mathcal{Q}\bar{\mathcal{Q}}$ splittings

$p_i(x, t) =$ probability that at "time" t, the flavor i carries a momentum fraction x at angular scale Δ

$$t = \int_{Q^2\Delta^2}^{Q^2\Delta_0^2} \frac{dq_t^2}{q_t^2} \frac{\alpha_S^{\mathrm{CMW}}(q_t^2)}{2\pi}$$

Collinear time that encodes the angular dependence,

Δ_0 intial splitting angle

DGLAP-type evolution

$$\frac{d}{dt}p_i(x,t) = \int_0^1 \frac{dz}{z}\left[P_{ij}^{(\mathrm{R})}p_j\left(\frac{x}{z},t\right) - zP_{ij}^{(\mathrm{V})}p_j(x,t)\right]$$

$P^{(R)}, P^{(V)}$ splitting kernels which impose that the leading parton is defined following the hardest branch.

The DGLAP type evolution equation is solved in two distinct regimes:

- $t < t_d$:

All flavors evolve with massless splitting functions

$$t_d = \int_{Q^2\Delta_d^2}^{Q^2\Delta_0^2} \frac{dq_t^2}{q_t^2}\frac{\alpha_S^{\mathrm{CMW}}(q_t^2)}{2\pi}, \quad \Delta_d = \frac{m}{Q}$$

- $t > t_d$:

Heavy flavor cannot be produce via gluon splittings and they cannot emit gluons at angles $\Delta < \Delta_d$. As a consequence we freeze p_Q and reduce the number of active flavors in the gluon splitting functions

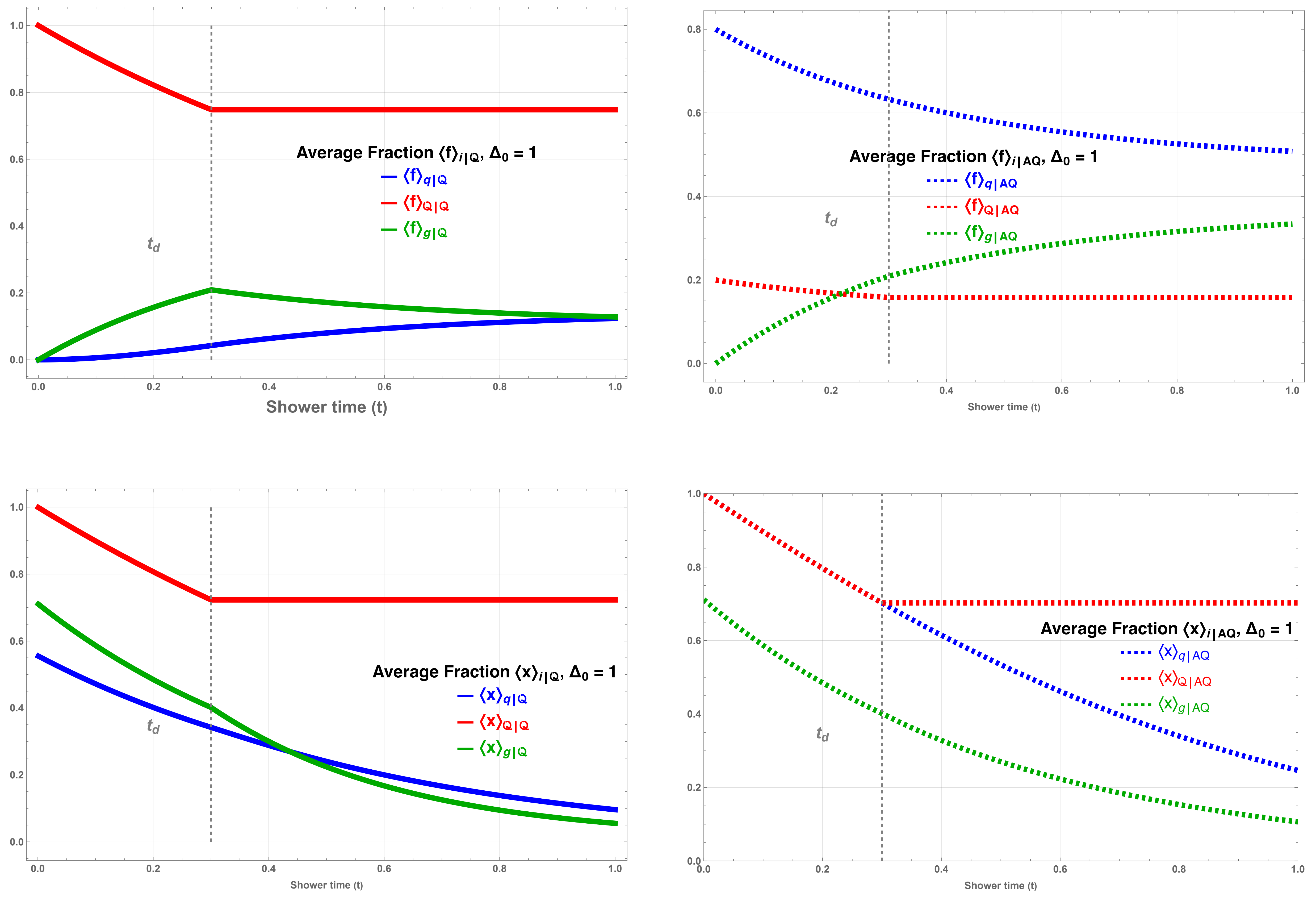
Moments of the probability distribution

It is interesting to consider the first two moments of the distribution:

- $f_i = \tilde{p}_i(1,t)$ average fraction of quarks/gluons at time t

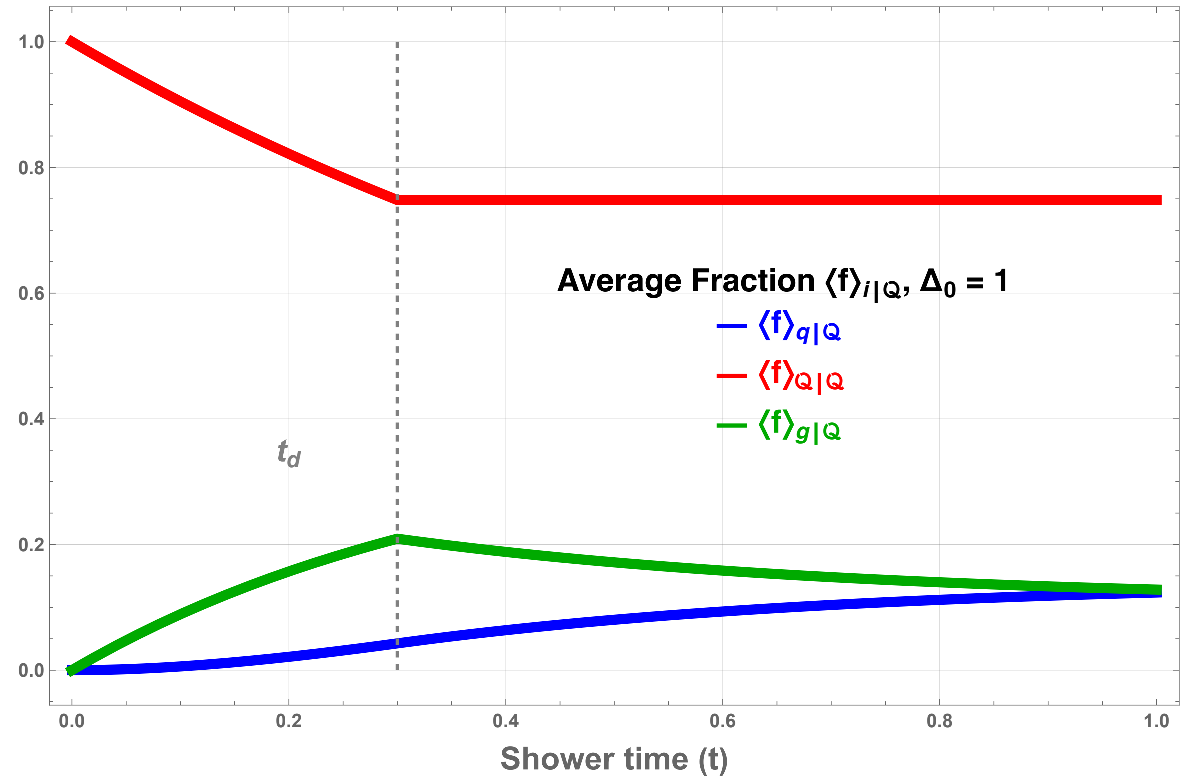
- $\bar{x}_i = \dfrac{\tilde{p}_i(2,t)}{f_i}$ average momentum fraction at time t

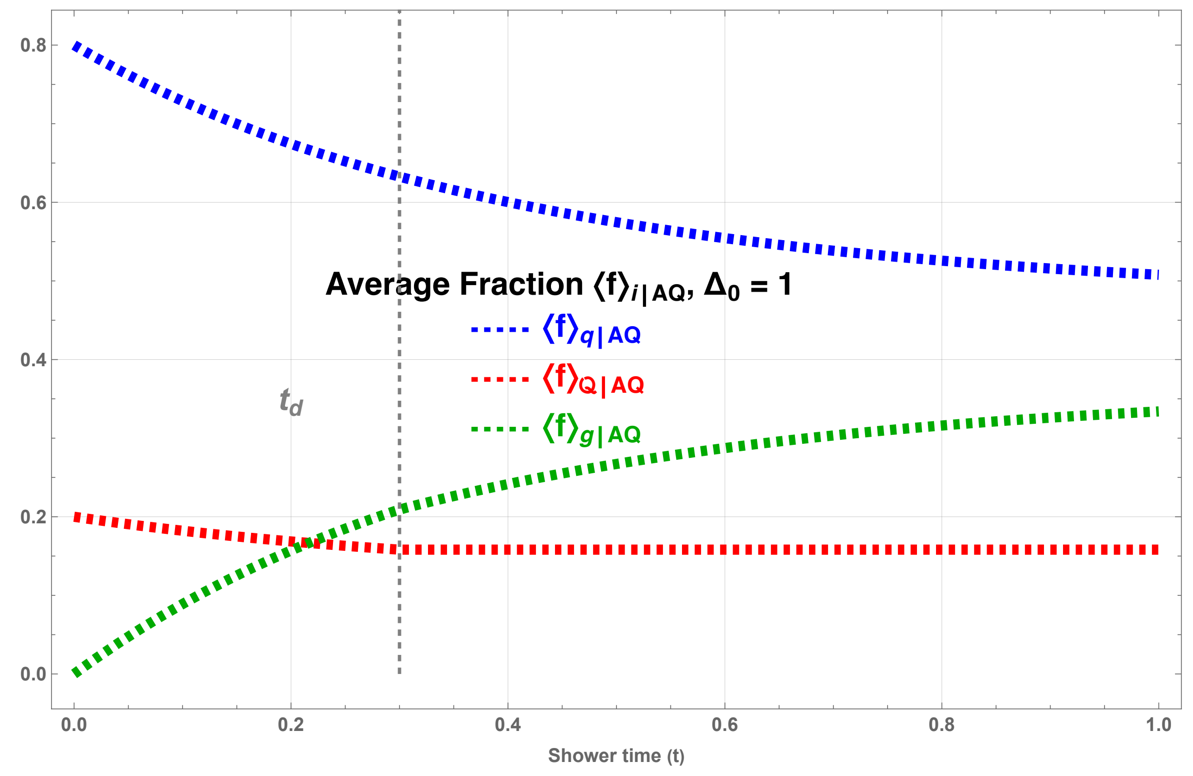
We examined both heavy quark jet production $(\,|\,\mathcal{Q})$ and inclusive jet production $(\,|\,\mathrm{AQ})$

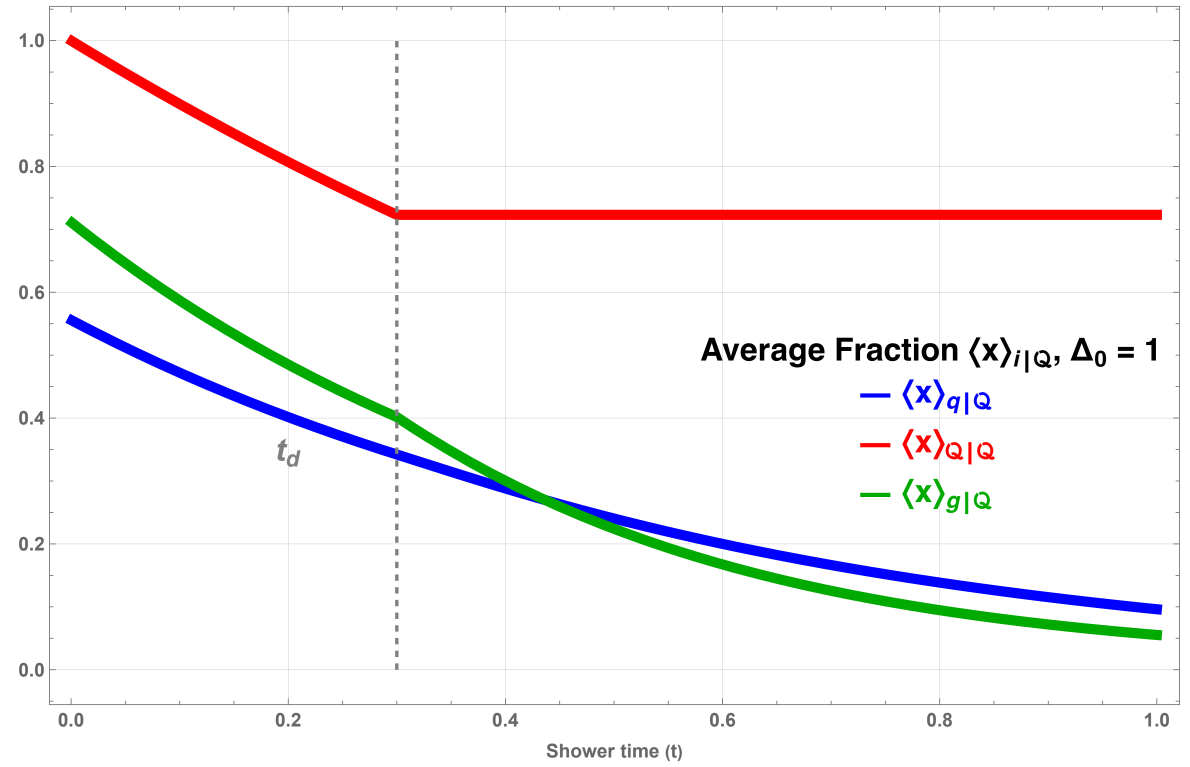


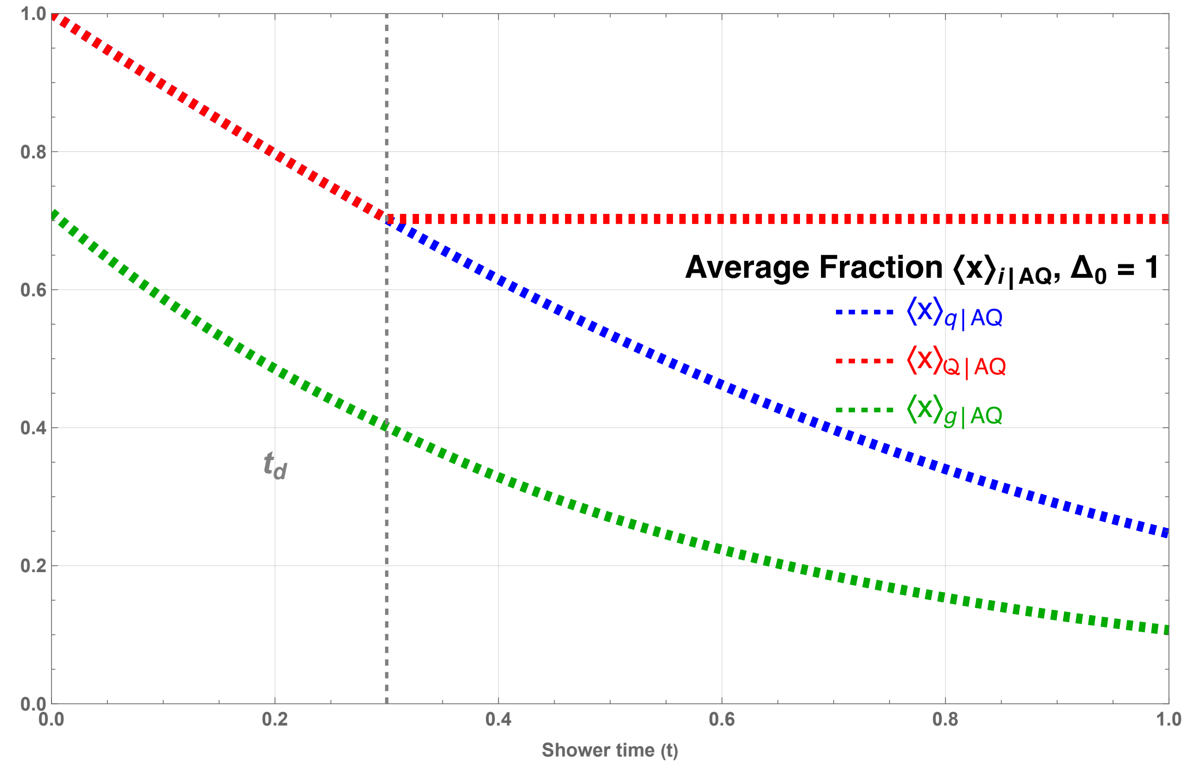
Moments of the probability distribution

- $f_i = \tilde{p}_i(1,t)$ average fraction of quarks/gluons at time t
- $\bar{x}_i = \dfrac{\tilde{p}_i(2,t)}{f_i}$ average momentum fraction at time t



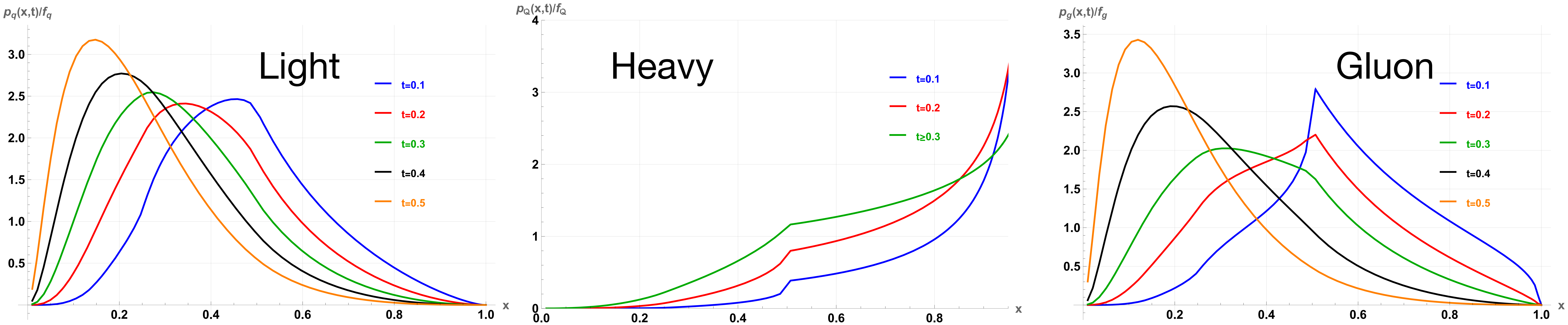






- Fractions of b quarks remains constant after t_d $\rightarrow$ no more gluon radiation off heavy quark
- Same for the average momentum fraction
- For $t < t_d$, $\bar{x}_q = \bar{x}_Q$: the jet originates from a quark, and the heavy flavour evolves identically to the light quarks up to t_d

Probability distributions in x-space



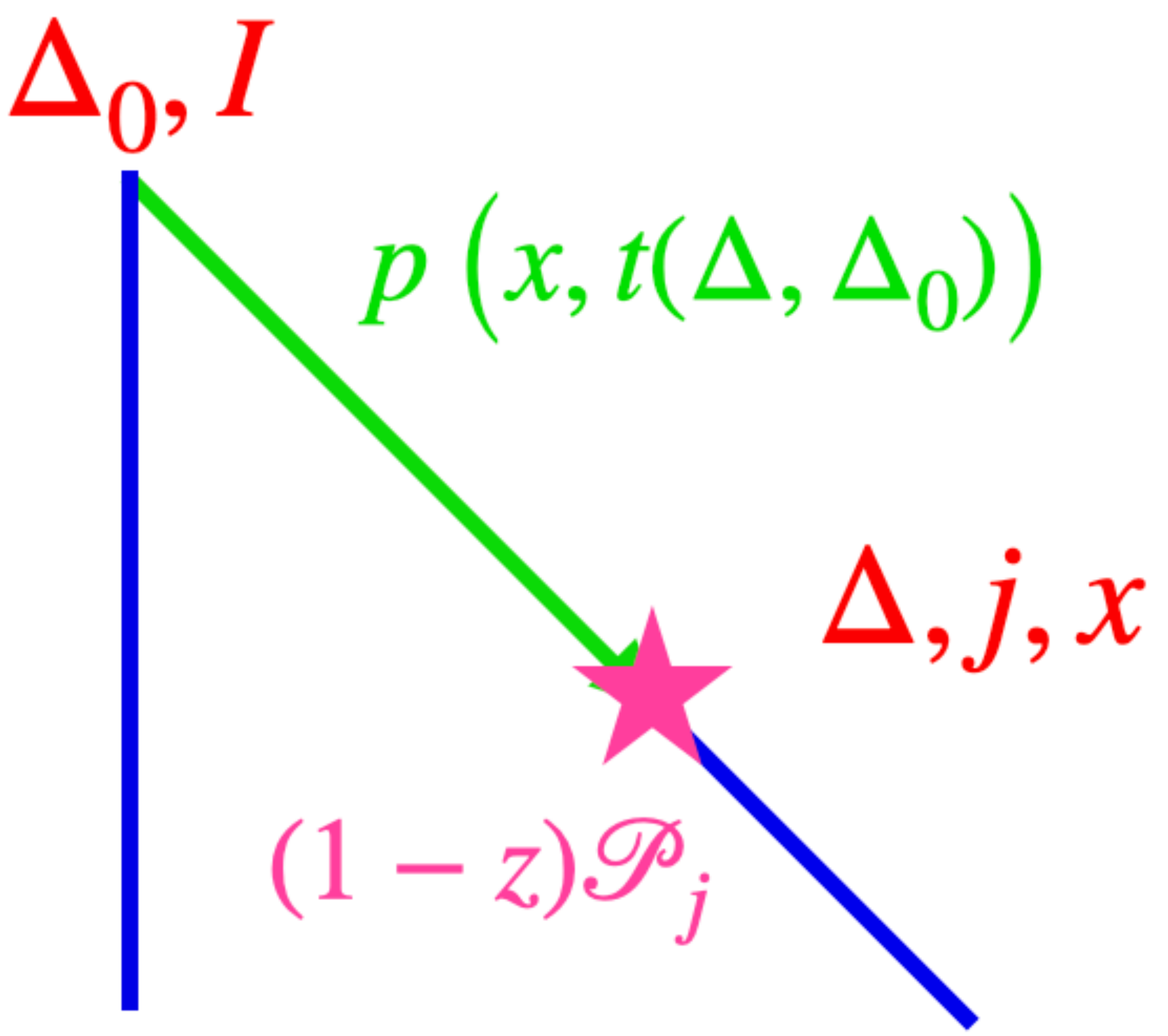
We consider a jet seeded by a HF

- As expected, at early time the heavy flavor component is sharply peaked at $x = 1$,
- The peak gradually smooths out as time progresses.

Collinear Lund plane density

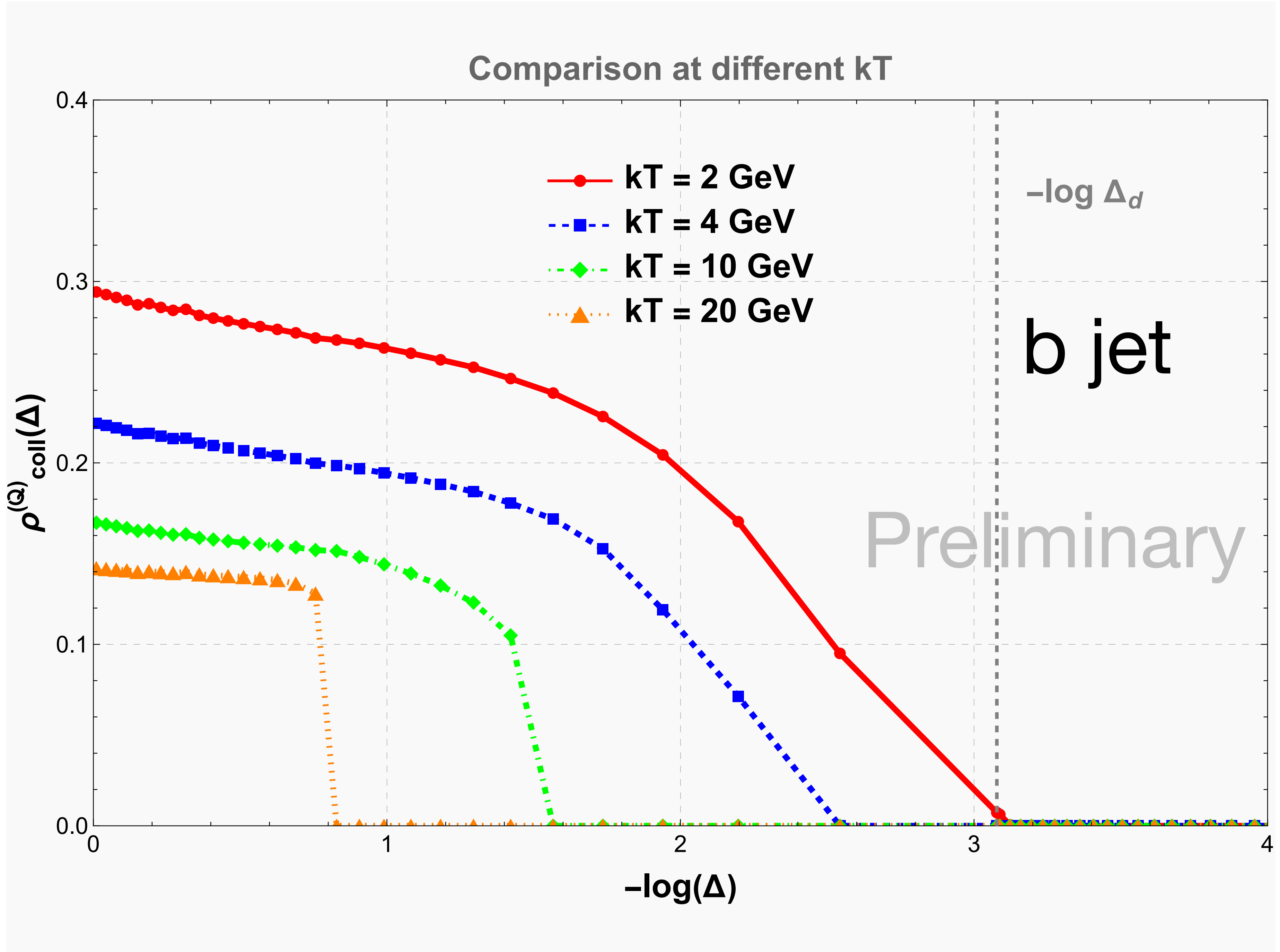
The collinear part of the Lund plane density (resummation of collinear logs) reads:

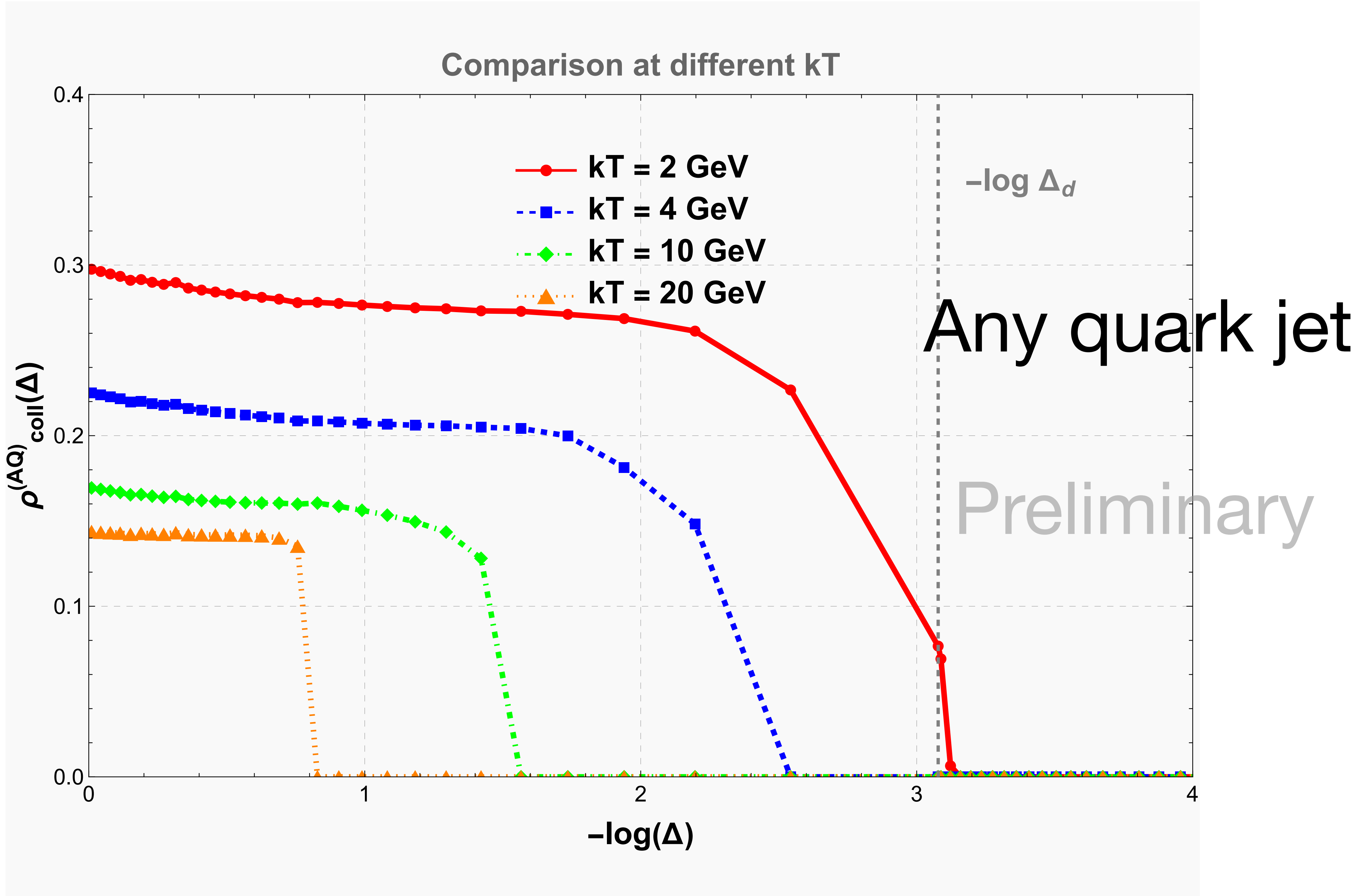
$$\rho_I^{\mathrm{coll}} = \sum_{j=q,\mathcal{Q},g} \frac{\alpha_S^{\mathrm{CMW}}(k_t^2)}{\pi} \int_0^1 dx\, p_j(x,t)(1-z)\mathscr{P}_j(z)\,|_{1-z=\frac{k_t}{xQ\Delta}}$$



- Evolution from Δ_0 to Δ. Then we sum over all possible flavors and over all possible energy fraction
- The properties of the last splitting are encoded into $\mathscr{P}_j$. We use here the full splitting function.

Collinear Lund plane density





- We considered both heavy quark jet and inclusive quark jet production.
- On the left, $p_{\mathbb{Q}}$ is the most relevant. The behavior at large Δ is governed by the quasi collinear splitting function. Suppression at small Δ due to dead-cone.
- On the right, the shape is more flat at large Δ.

Resummation of soft logarithms: $\mathcal{O}(\alpha_S^2)$ analysis

- We consider soft gluon emission in the collinear limit at commensurate angles:

$$\theta_1 \sim \theta_2 \sim \theta_{12} \ll 1$$

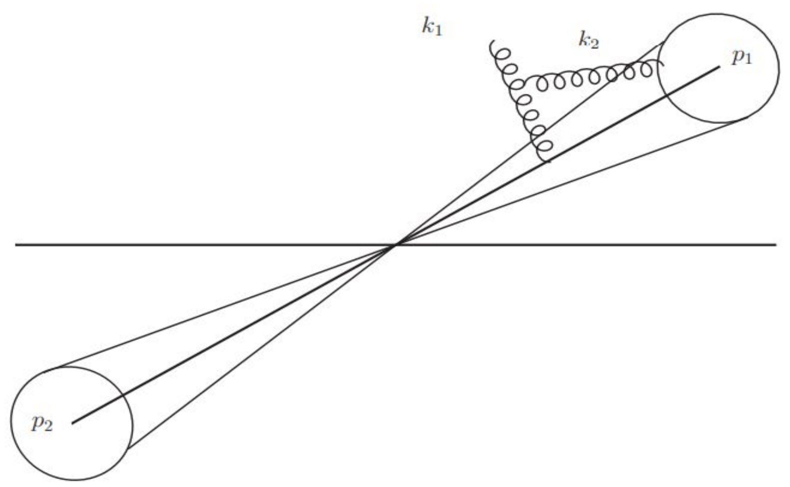
- Collinear limit is sufficient to catch the log enhancements
- Logs of k_t stem from non global configurations and C/A clustering effects
- In the collinear limit we can assign a flavor label to $\rho^{(\text{soft})}$

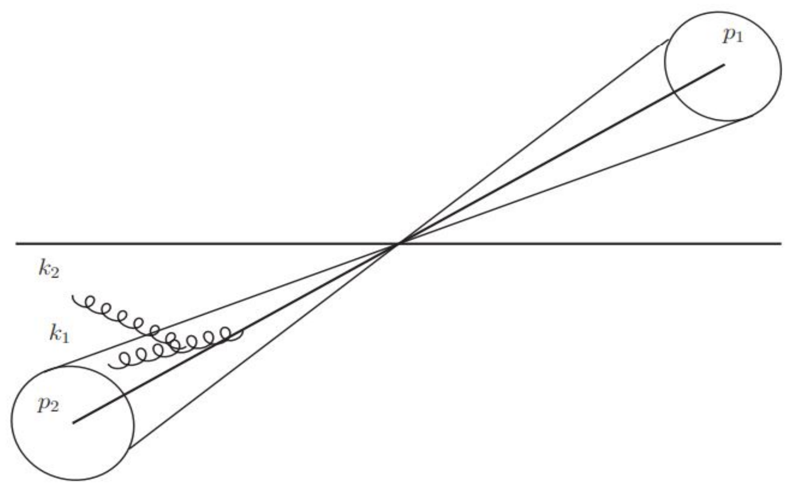
$$\rho_q^{(\text{soft})} = \frac{2\alpha_S C_F}{\pi}\left(1 + \frac{2\alpha_S}{\pi}(C_F - C_A)0.3\log\frac{k_t}{Q\Delta} + \mathcal{O}\left(\alpha_S^3\right)\right) = \frac{2\alpha_S C_F}{\pi}\bar{\rho}_q^{(\text{soft})}$$

$$\rho_{\mathcal{Q}}^{(\text{soft})} = \frac{2\alpha_S C_F}{\pi}\left(1 + \frac{2\alpha_S}{\pi}\left(\rho^{(\text{soft})}_{\mathcal{Q},C_F^2} - \rho^{(\text{soft})}_{\mathcal{Q},C_F C_A}\right)\log\frac{k_t}{Q\Delta} + \mathcal{O}\left(\alpha_S^3\right)\right)$$

$$= \frac{2\alpha_S C_F}{\pi}\left(\frac{\Delta^2}{\Delta^2 + \xi}\right)^2 \bar{\rho}_{\mathcal{Q}}^{(\text{soft})}$$

$$\rho_g^{(\text{soft})} = \frac{2\alpha_S C_A}{\pi}\left(1 + \mathcal{O}\left(\alpha_S^3\right)\right) = \frac{2\alpha_S C_A}{\pi}\bar{\rho}_g^{(\text{soft})}$$





Form factors scaling

It is interesting to study the behavior of the two form factors $\rho^{(\mathrm{soft})}_{\mathcal{Q},C_F^2}, \rho^{(\mathrm{soft})}_{\mathcal{Q},C_F C_A}$ in the limit $\Delta^2 \ll \xi$

- The abelian contribution to $\rho^{(\mathrm{soft})}_{\mathcal{Q}}$ scales overall as

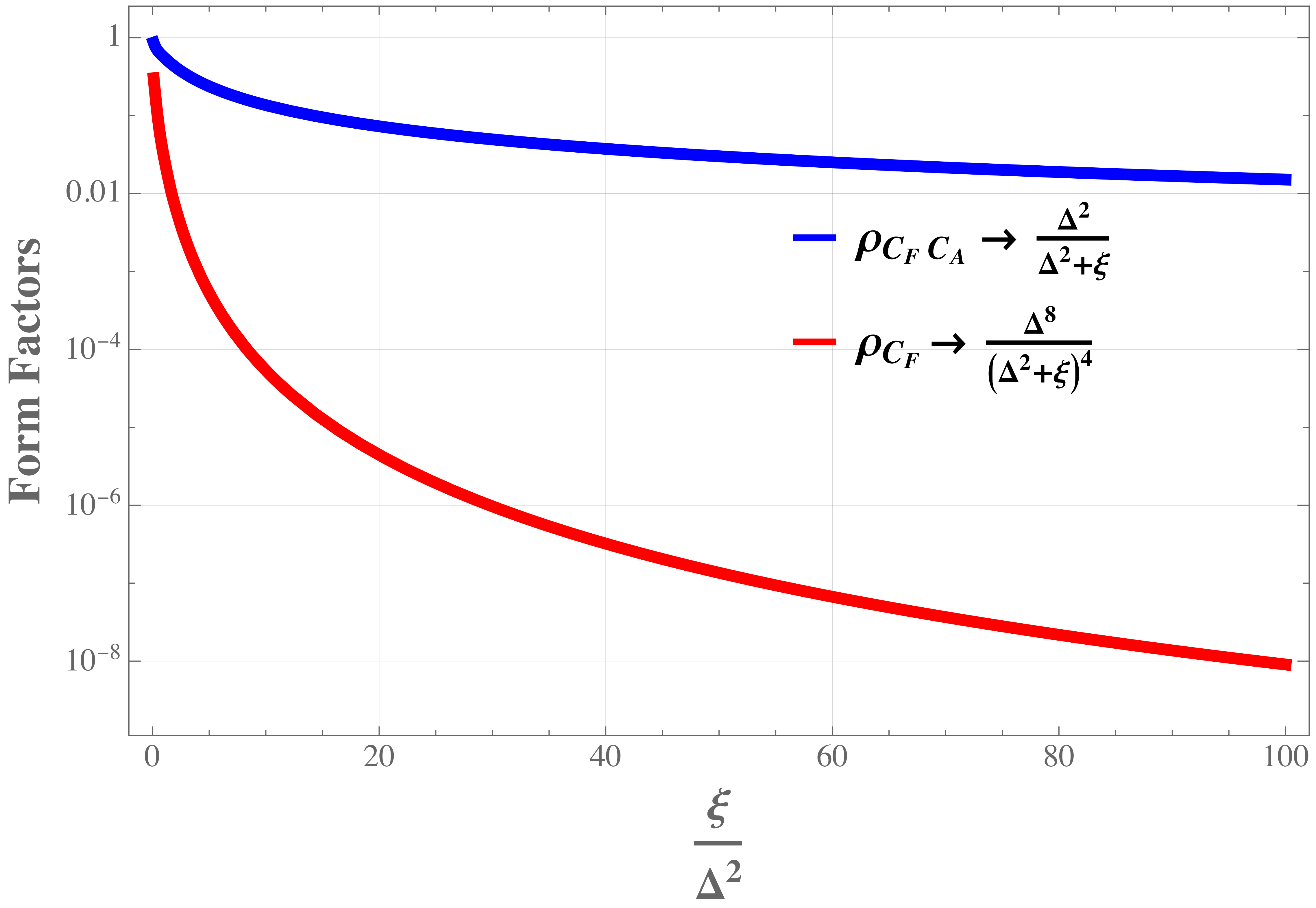
$$\rho^{(\mathrm{soft})}_{\mathcal{Q},C_F^2} \sim \left(\frac{\Delta^2}{\Delta^2+\xi}\right)^4$$

$\rightarrow$ double suppression

- The non-abelian contribution to $\rho^{(\mathrm{soft})}_{\mathcal{Q}}$ scales overall as

$$\rho^{(\mathrm{soft})}_{\mathcal{Q},C_F C_A} \sim \left(\frac{\Delta^2}{\Delta^2+\xi}\right)$$

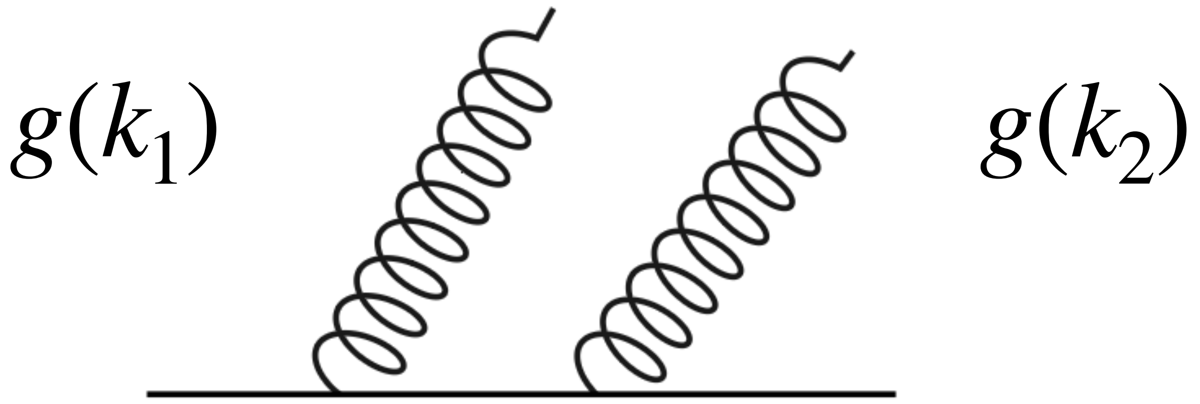
$\rightarrow$ less suppressed than one emission !!



Form factors scaling

We inspect closer the structure of the form factors

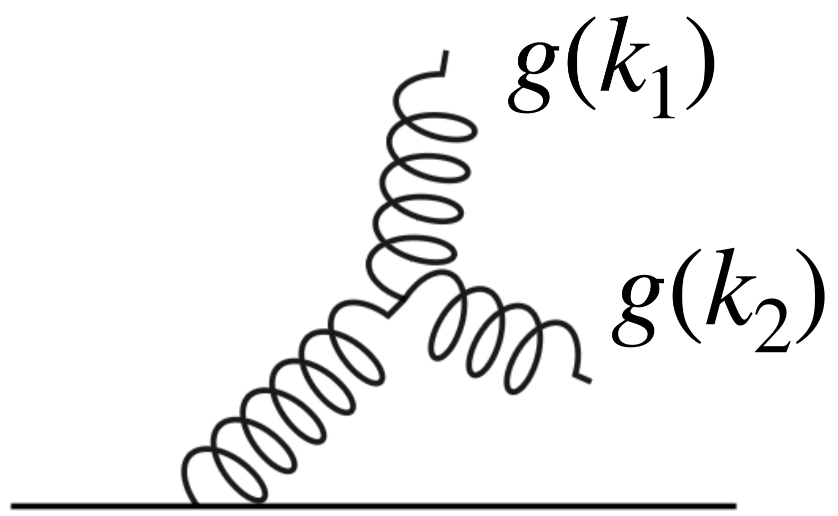
- Abelian contribution



$$= \left(\frac{\Delta^2}{\Delta^2+\xi}\right)^2 \times \left(\frac{\Delta^2}{\Delta^2+\xi}\right)^2$$

As expected: both gluons feel the dead-cone

- Non-Abelian contribution



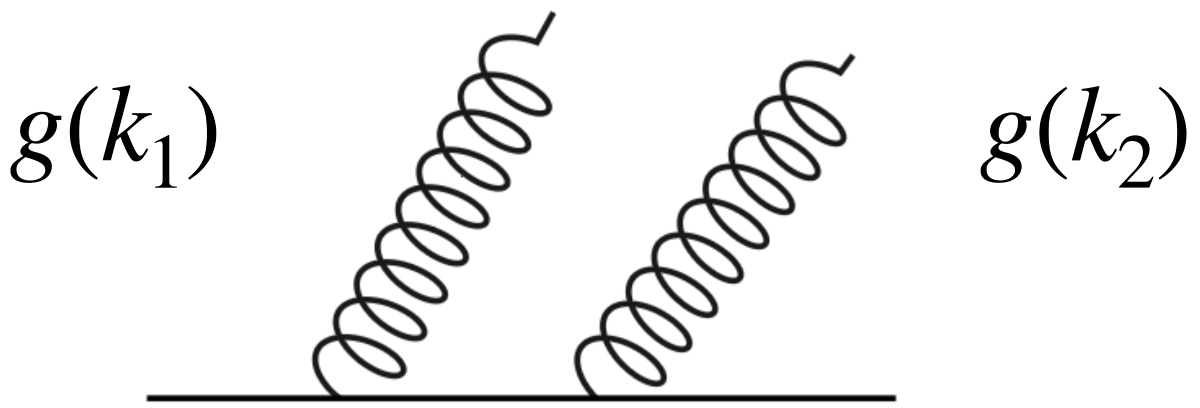
$$\neq 1 \times \left(\frac{\Delta^2}{\Delta^2+\xi}\right)^2$$

→ Expected behavior: first gluon does not feel the dead cone.

Form factors scaling

We inspect closer the structure of the form factors

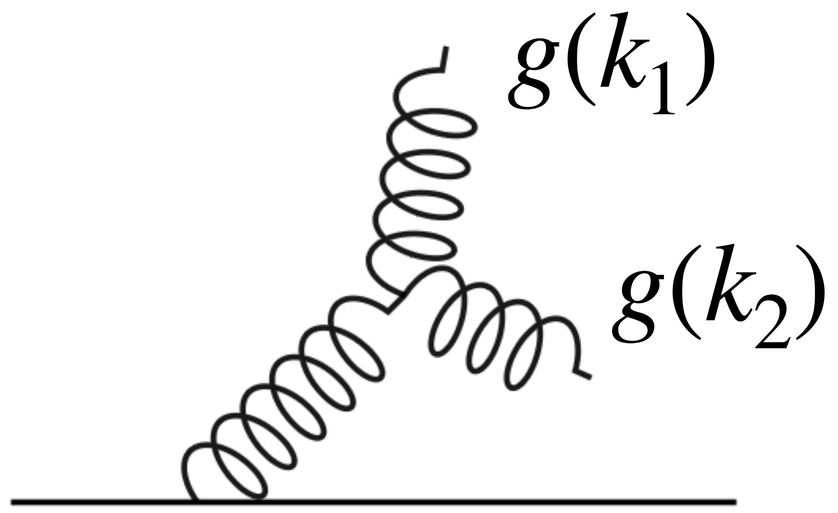
- Abelian contribution



$$= \left(\frac{\Delta^2}{\Delta^2 + \xi}\right)^2 \times \left(\frac{\Delta^2}{\Delta^2 + \xi}\right)^2$$

→As expected: both gluons feel the dead-cone

- Non-Abelian contribution



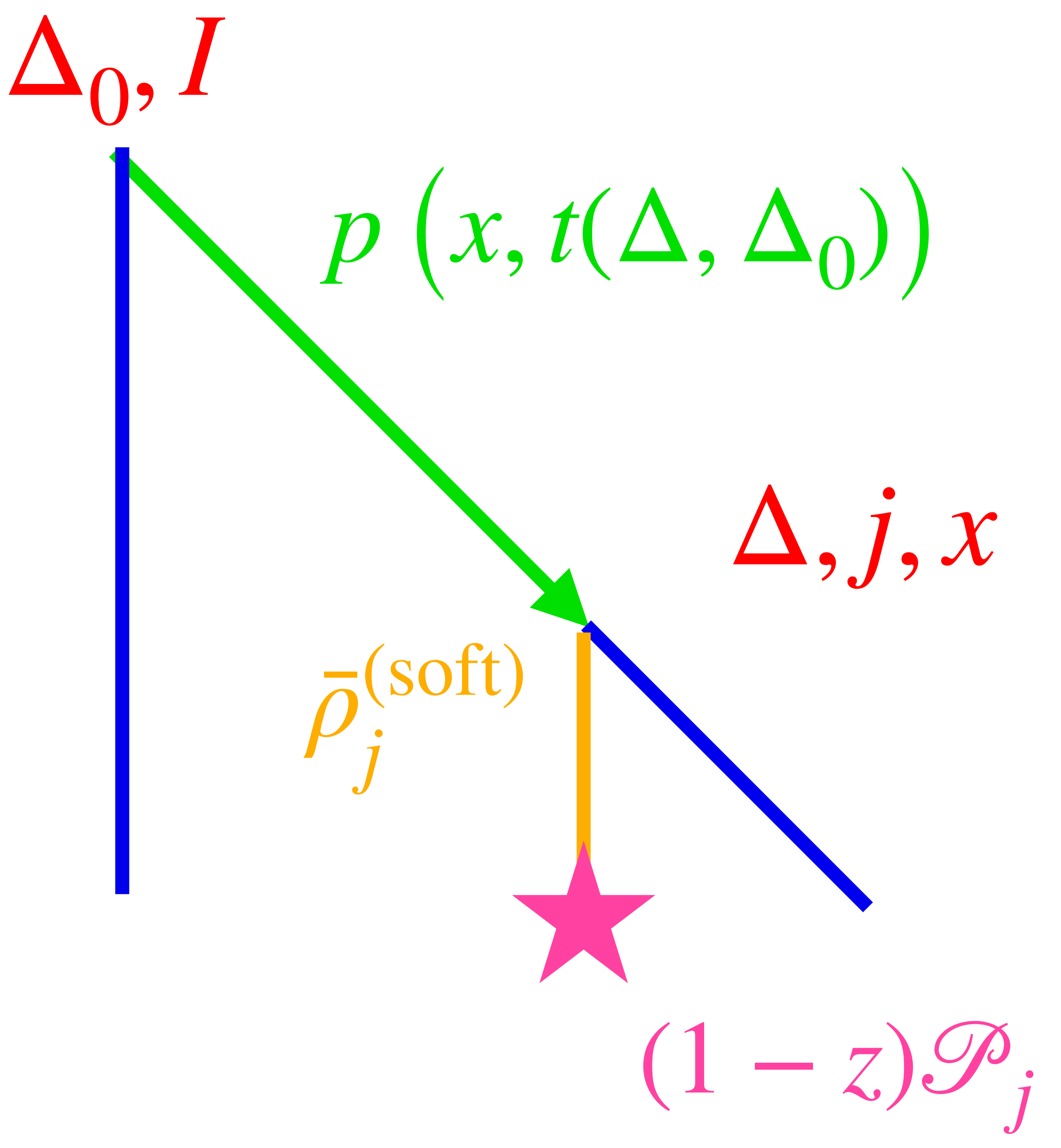
$$= \left(\frac{\Delta^2}{\Delta^2 + \xi}\right)$$

→ Dominated by configuration where the emission at angle $\theta_1 \simeq \xi$ favors emissions deep inside the dead-cone

Resummation of the Lund plane density

- The (almost) complete resummation of the primary Lund plane density is given by:

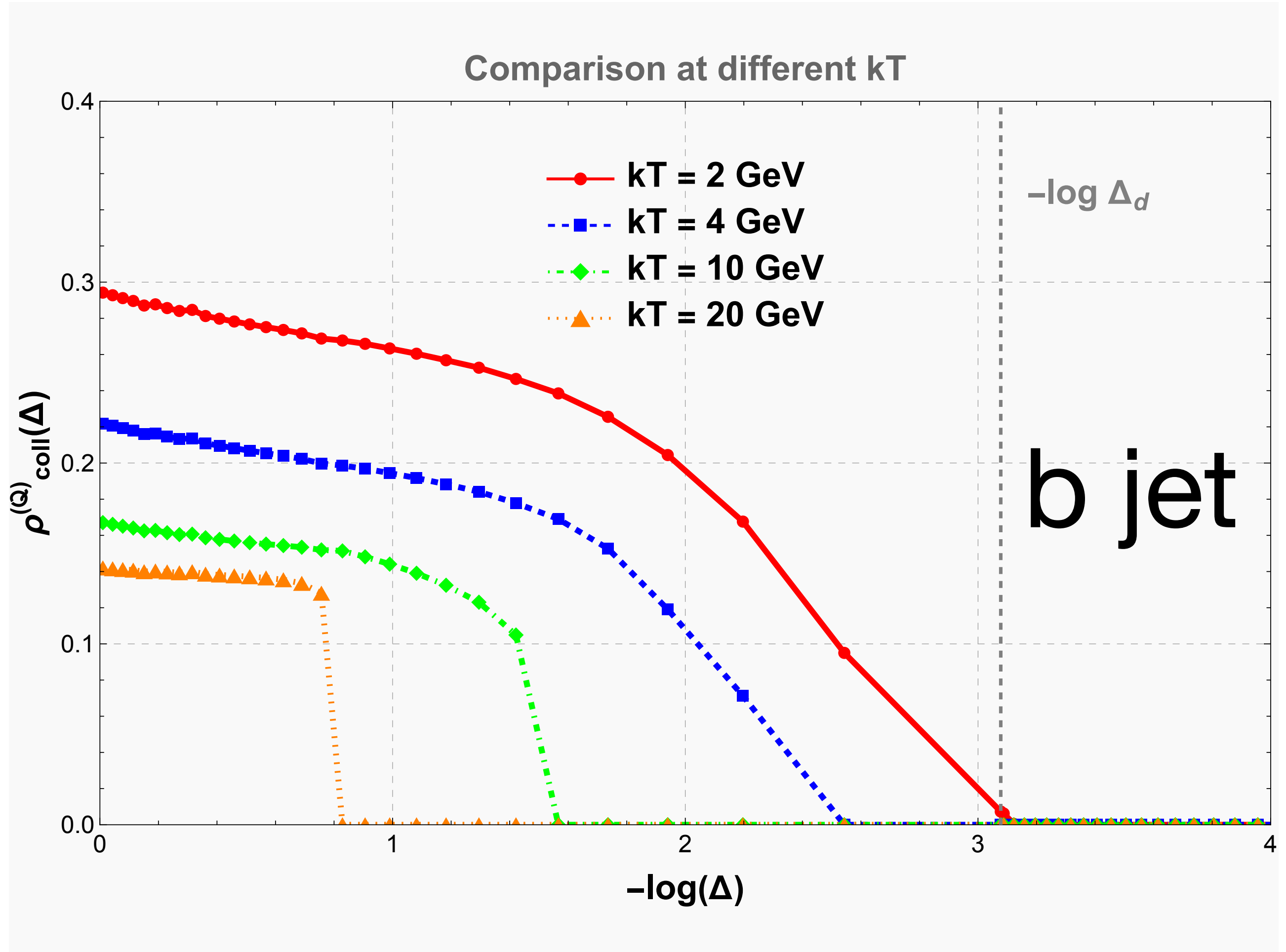
$$\rho_I^{\text{soft}-\text{coll}} = \sum_{j=q,\mathcal{Q},g} \frac{\alpha_S^{\text{CMW}}(k_t^2)}{\pi} \int_0^1 dx\, p_j(x,t)(1-z)\mathscr{P}_j(z)\,|_{1-z=\frac{k_t}{xQ\Delta}}\, \bar{\rho}_j^{(\text{soft})}$$

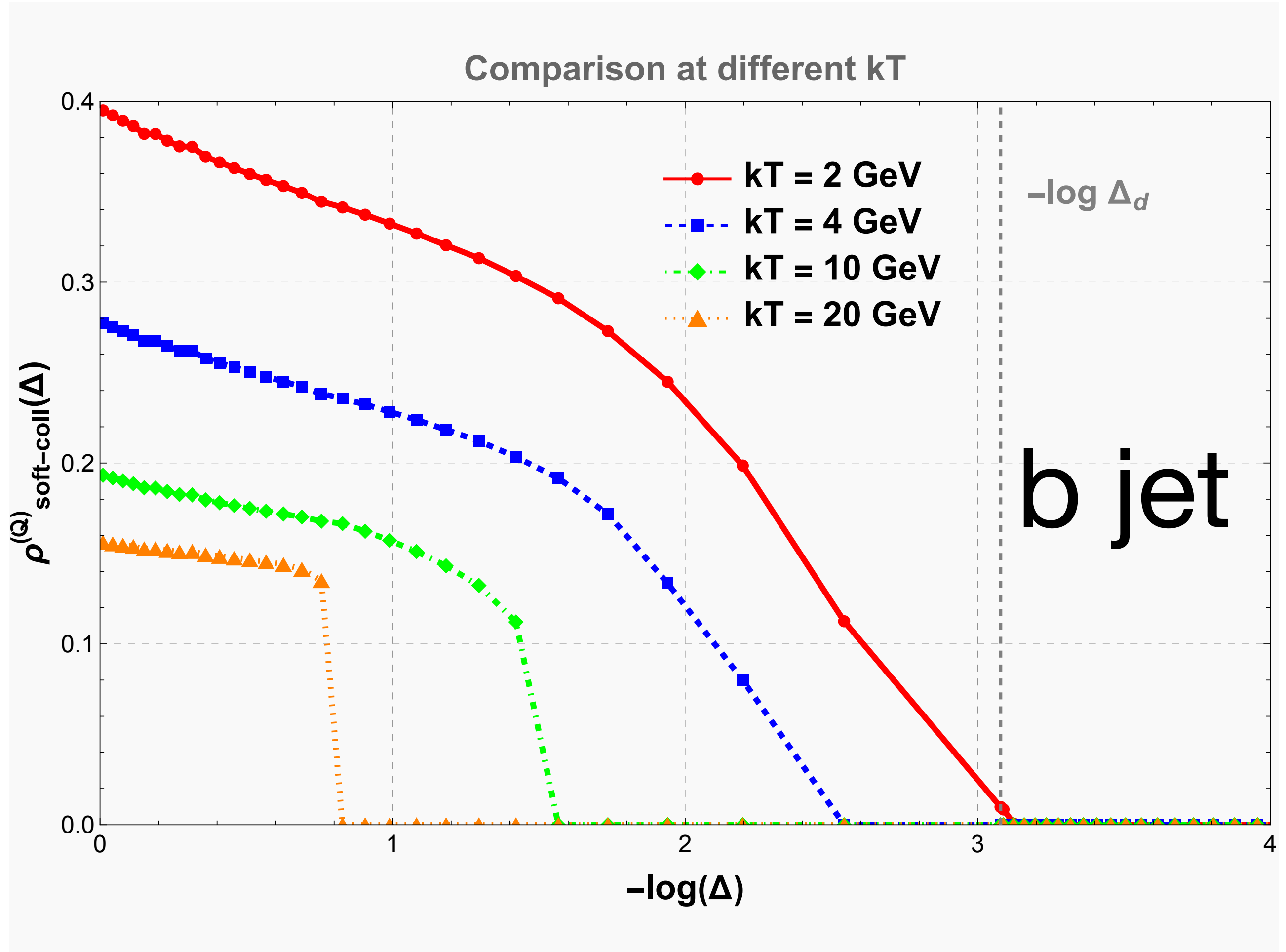


- The soft part of the Lund density evolves at fixed Δ from xQ down to k_t.
- The all-order inclusion of soft logs is still work in progress
- $\rho^{(\text{soft})}$ is considered in the collinear/quasi-collinear limit at order $\mathcal{O}(\alpha_S^2)$

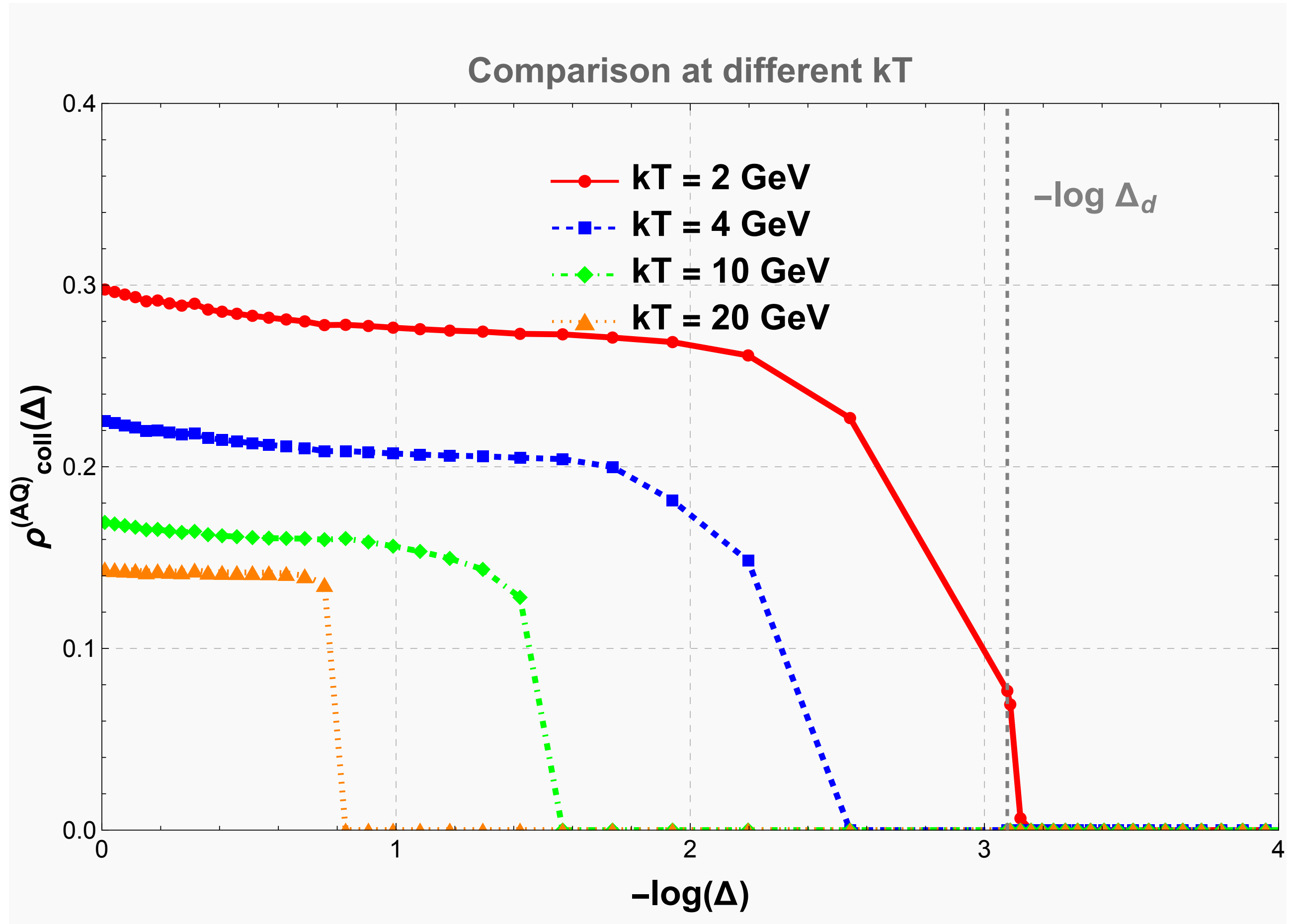
Resummation of the Lund plane density

- We show the comparison between ρ_I^{coll} and $\rho_I^{\text{soft-coll}}$ to gauge the impact of soft resummation.
- Enhancement in the large Δ region from logs of k_t
- At small values of Δ, the effect of C/A clustering causes the curve to rise.
- The effect of soft resummation is reduced at larger values of k_t

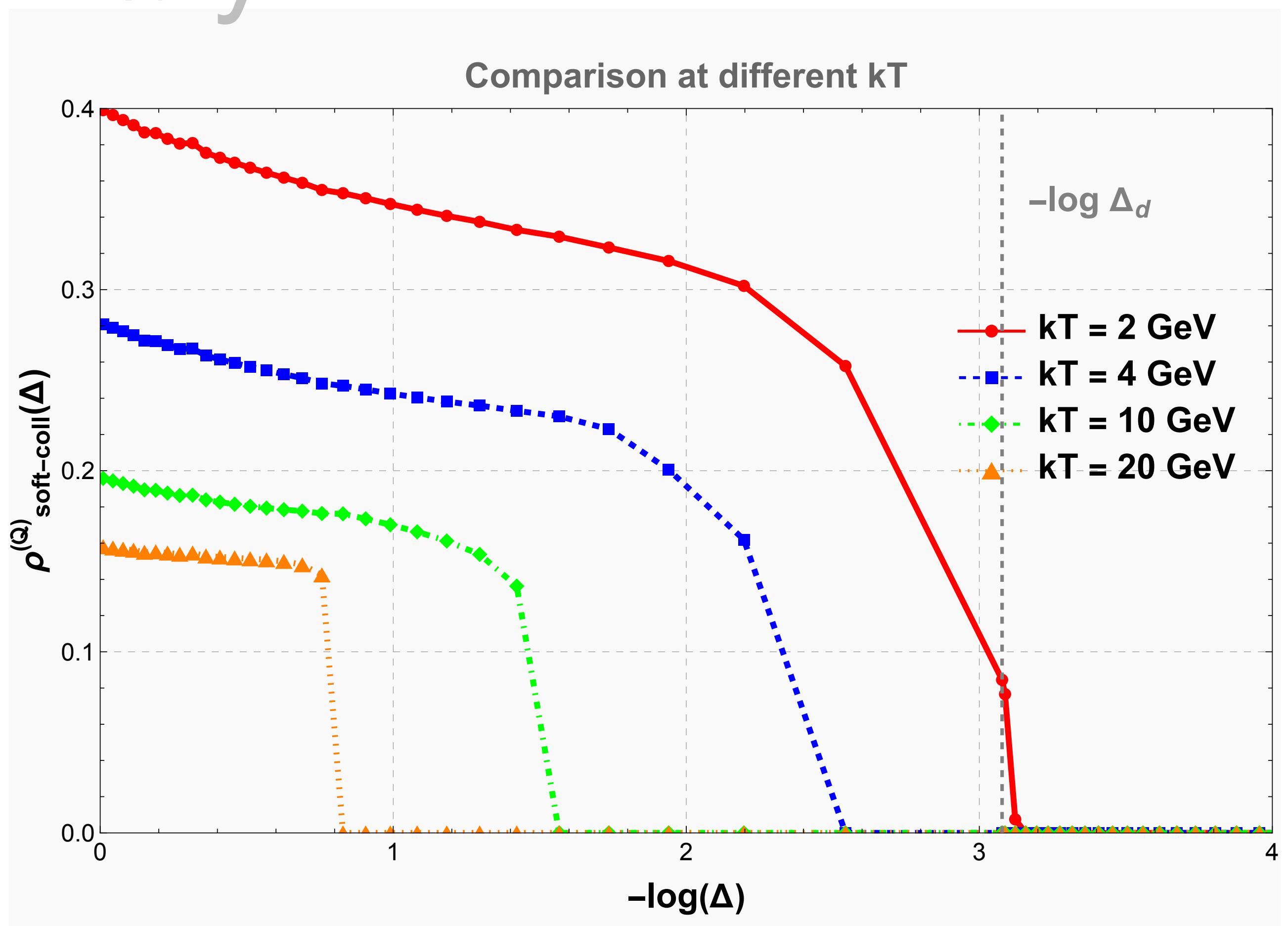




Preliminary

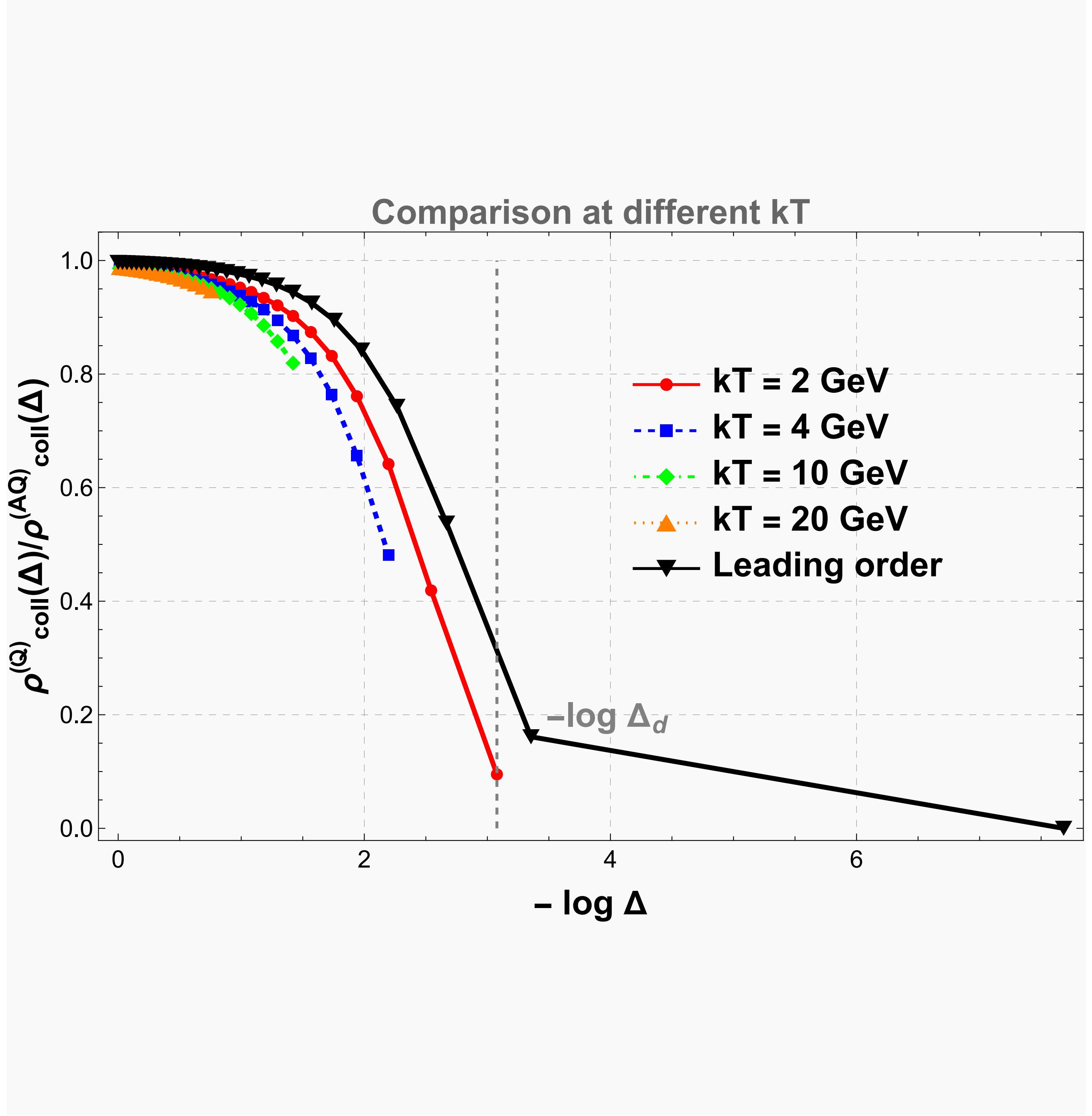


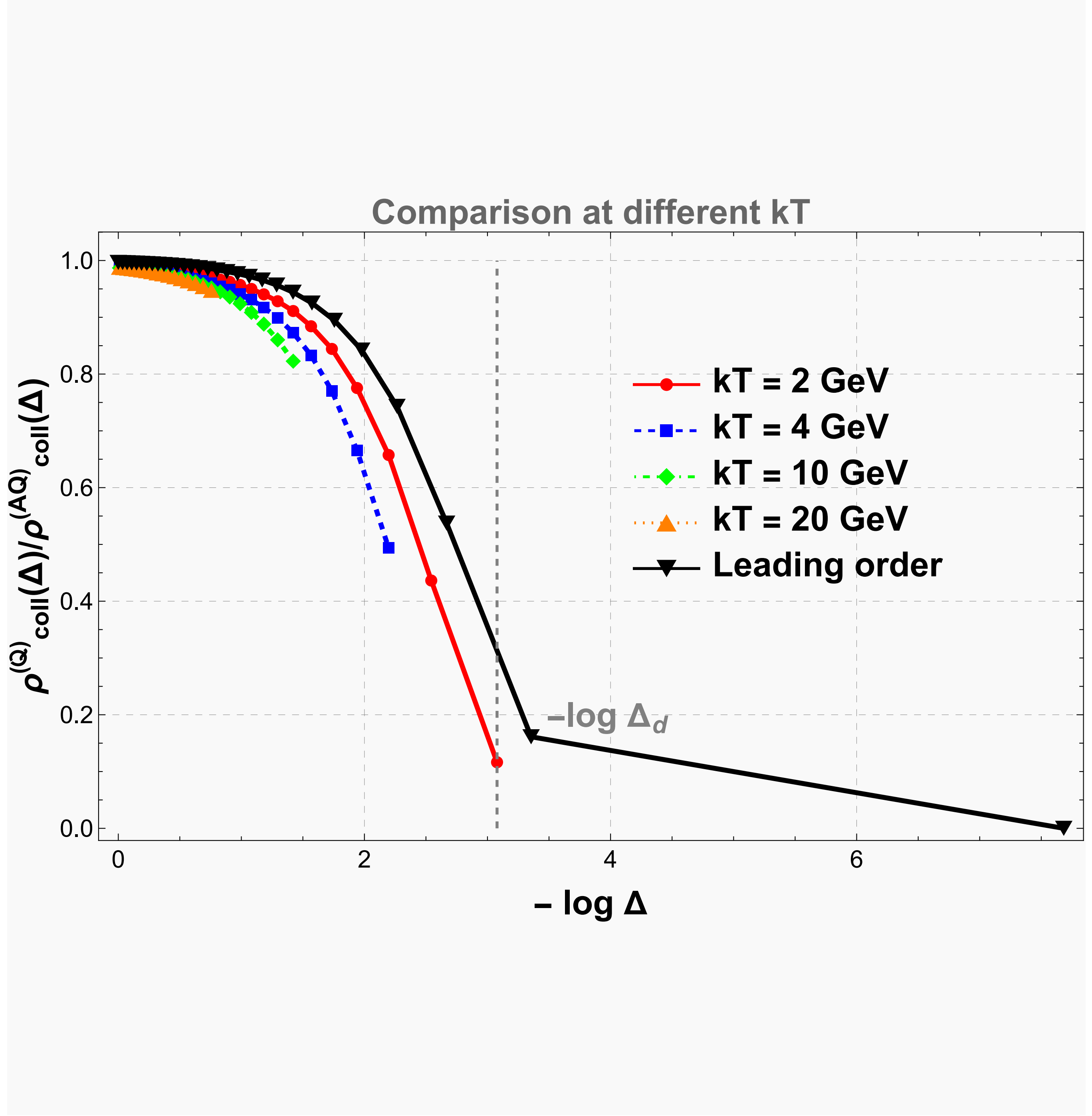
Any quark jet

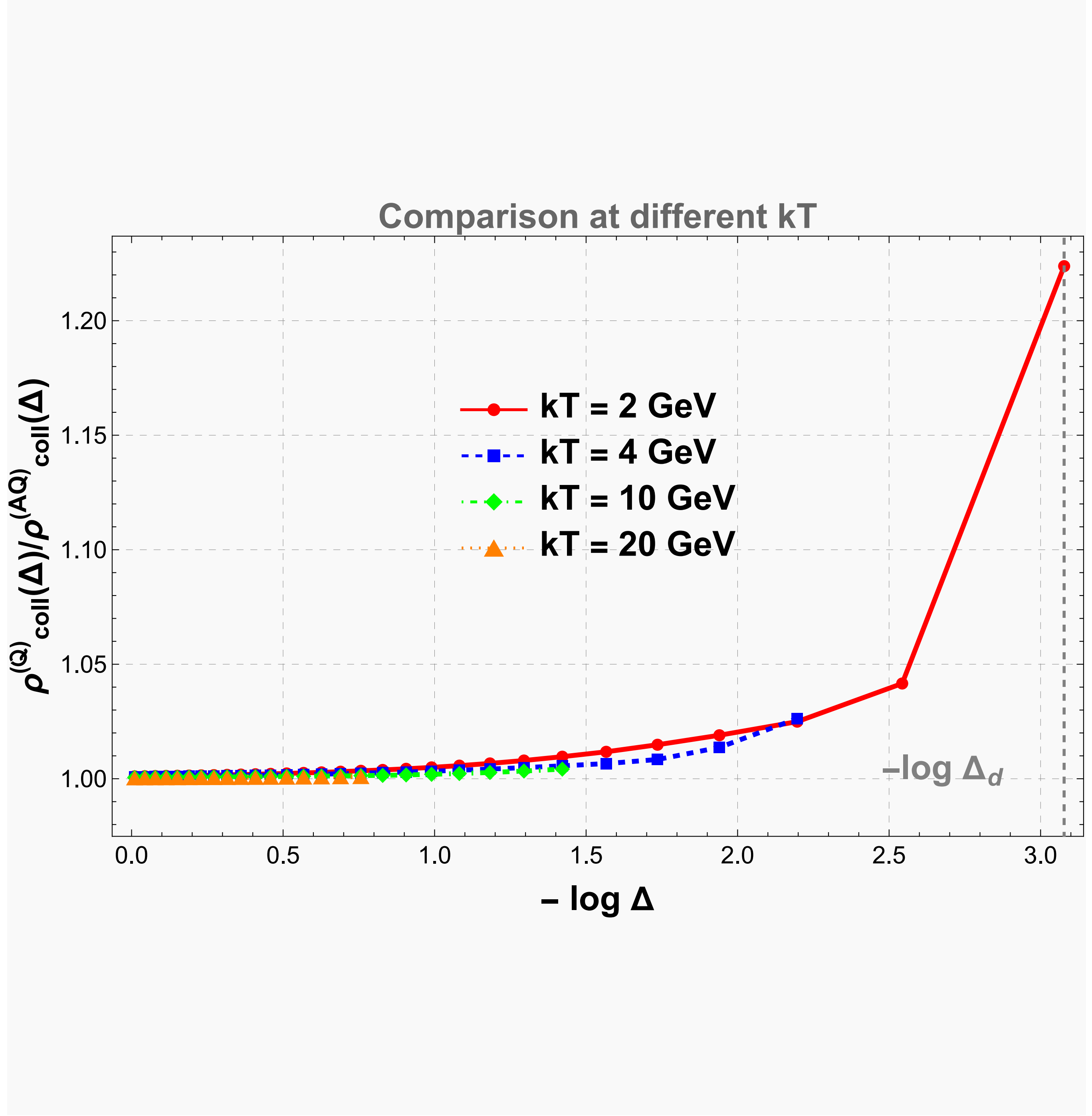


Any quark jet

Ratio plots







- We plot the ratio of collinear and soft-collinear densities against leading order calculation at different k_t.
- Collinear evolution causes a further suppression wrt the leading order.
- The last plot on the right displays the ratio of the other two plots. The observed behavior stems from clustering and non global logs.

Conclusions and outlook

- Lund plane density is a powerful tool to analyze jet subtructure being very sensitive to mass effect.
- Multiple resummations involved: $\log \Delta, \log k_t, \log \xi$
- Missing full resummation of $\rho^{(\text{soft})}$ and matching to fixed order.
- Examine gluon Lund plane density to study $g \to b\bar{b}$ splitting

Thanks for your attention !!!

Backup slides

Solution in Mellin space

- The expression for p_i can be obtained either by directly integrating the DGLAP equation or by performing a Mellin transform followed by numerical inversion to $x-$space (Fixed Talbot and GWR algorithms)

$$\frac{d}{dt}\tilde{p}_i(N,t) = \Gamma_{ij}\tilde{p}_j(N,t), \quad \Gamma_{ij} = \gamma_{ij} - \delta\gamma_{ij}$$

γ_{ij} = standard AP anomalous dimension

$\gamma_{ij} - \delta\gamma_{ij}$ = WTA moments of the splitting functions

S. Caletti et al.

- We examine two different case of initial condition (possible scenarios at LEP):

$$p_i(x,0) = \delta(1-x)\delta_{iQ}, \quad p_i(x,0) = \frac{4}{5}\delta(1-x)\delta_{iq} + \frac{1}{5}\delta(1-x)\delta_{iQ}$$

Jet tagged as heavy flavor at $t = 0$

Inclusive quark jet production at $t = 0$

Mellin inversion

The DGLAP type evolution equation can also be written as:

$$\frac{d}{dt} p_i(x,t) = \sum_j \int_x^1 \frac{dz}{z} \left(\hat{P}_{ij}(z) - \delta\hat{P}_{ij}(z) \right) p_j\left(\frac{x}{z}, t\right)$$

$\hat{P}_{ij}(z)$ are the distributional massless splitting functions.

$$\delta\hat{P}_{ij}(z) = \hat{P}_{ij}(z)\Theta(1-2z)$$

The evolution is splitted into two parts:

For $x > 1/2$ we get the standard DGLAP as: $\int_x^1 \frac{dz}{z} \delta\hat{P}_{ij}(z) p_j = 0$

In this region, both Fixed Talbot and GWR algorithms invert from Mellin to $x-$space, giving the same result

Mellin inversion

The DGLAP type evolution equation can also be written as:

$$\frac{d}{dt}p_i(x,t) = \sum_j \int_x^1 \frac{dz}{z}\left(\hat{P}_{ij}(z) - \delta\hat{P}_{ij}(z)\right) p_j\left(\frac{x}{z}, t\right)$$

$\hat{P}_{ij}(z)$ are the distributional massless splitting functions.

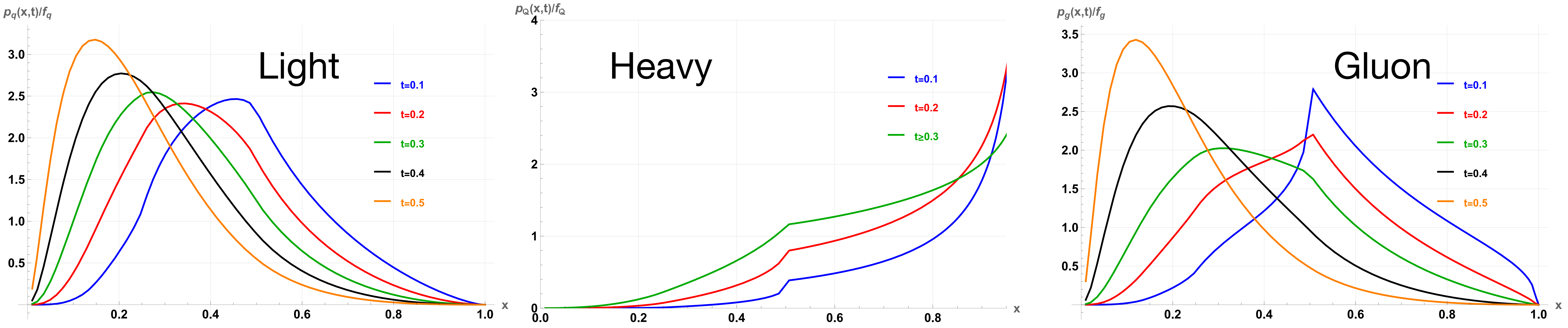
$$\delta\hat{P}_{ij}(z) = \hat{P}_{ij}(z)\Theta(1-2z)$$

The evolution is splitted into two parts:

For $x < 1/2$ the contribution of $\delta\hat{P}_{ij}(z)$ is different from 0.

- In this region, the Mellin space solution does not converge in the Talbot contour, due to the fact that for large negative $\mathscr{R}\{N\}$ the anomalous dimensions behave as $\delta\gamma_{ij} \sim 2^{-N}$
- In this region, only methods based on accelerated series get sensible results.

Probability distributions in x-space



Probability distribution obtained through Mellin inversion: highly non trivial due to non convergence of the integrand in the Talbot integration contour (more in the backup)

- Light quark and gluon distributions become enhanced in the small x region at large times
- Light quarks and gluons produced at large times have a small fraction of energy.