BOOST2025 @ 31.Jul.2025

First Integration of BSM radiations in Herwig7

[JHEP 08 (2024) 064]

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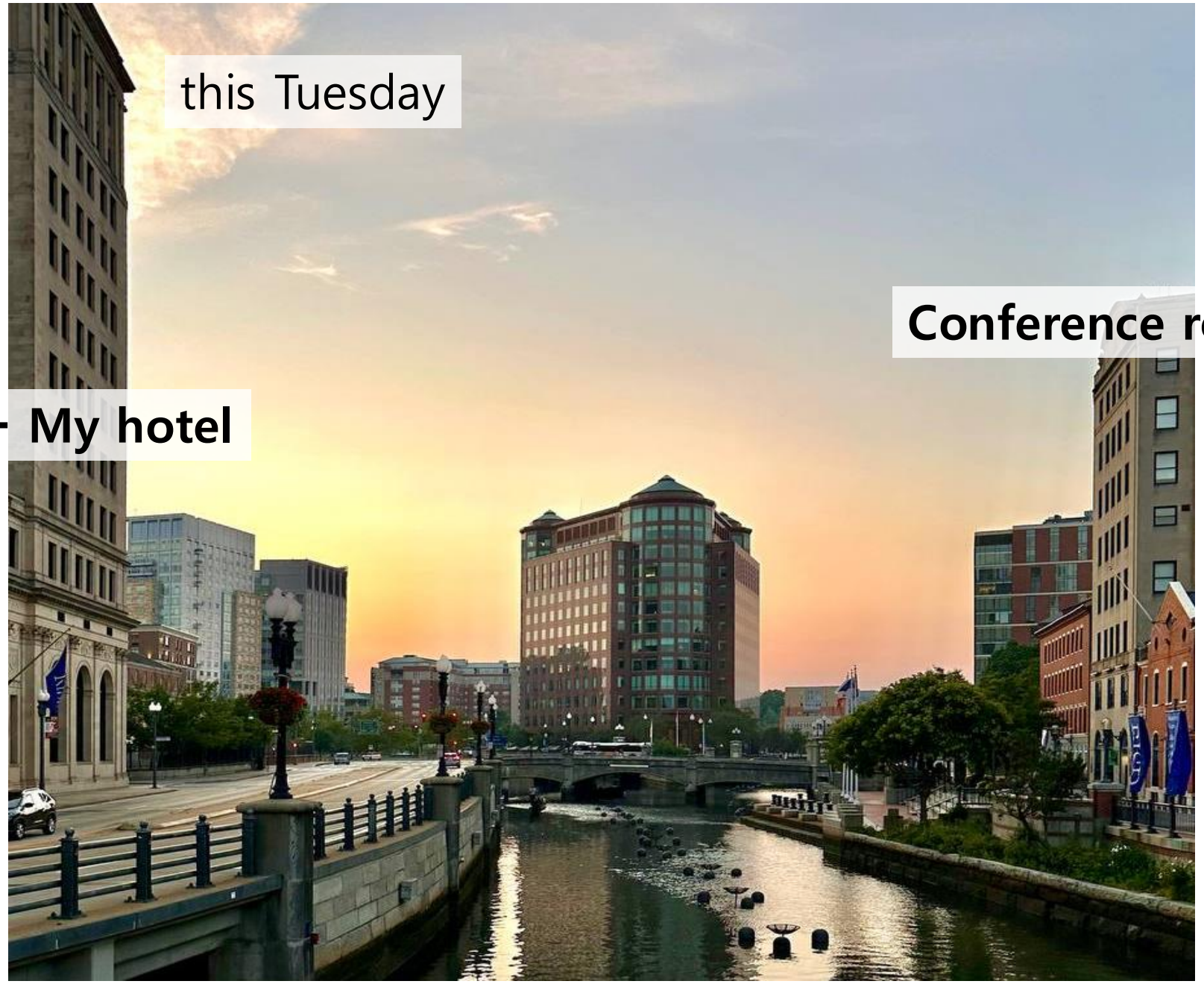
IPPP, Durham University

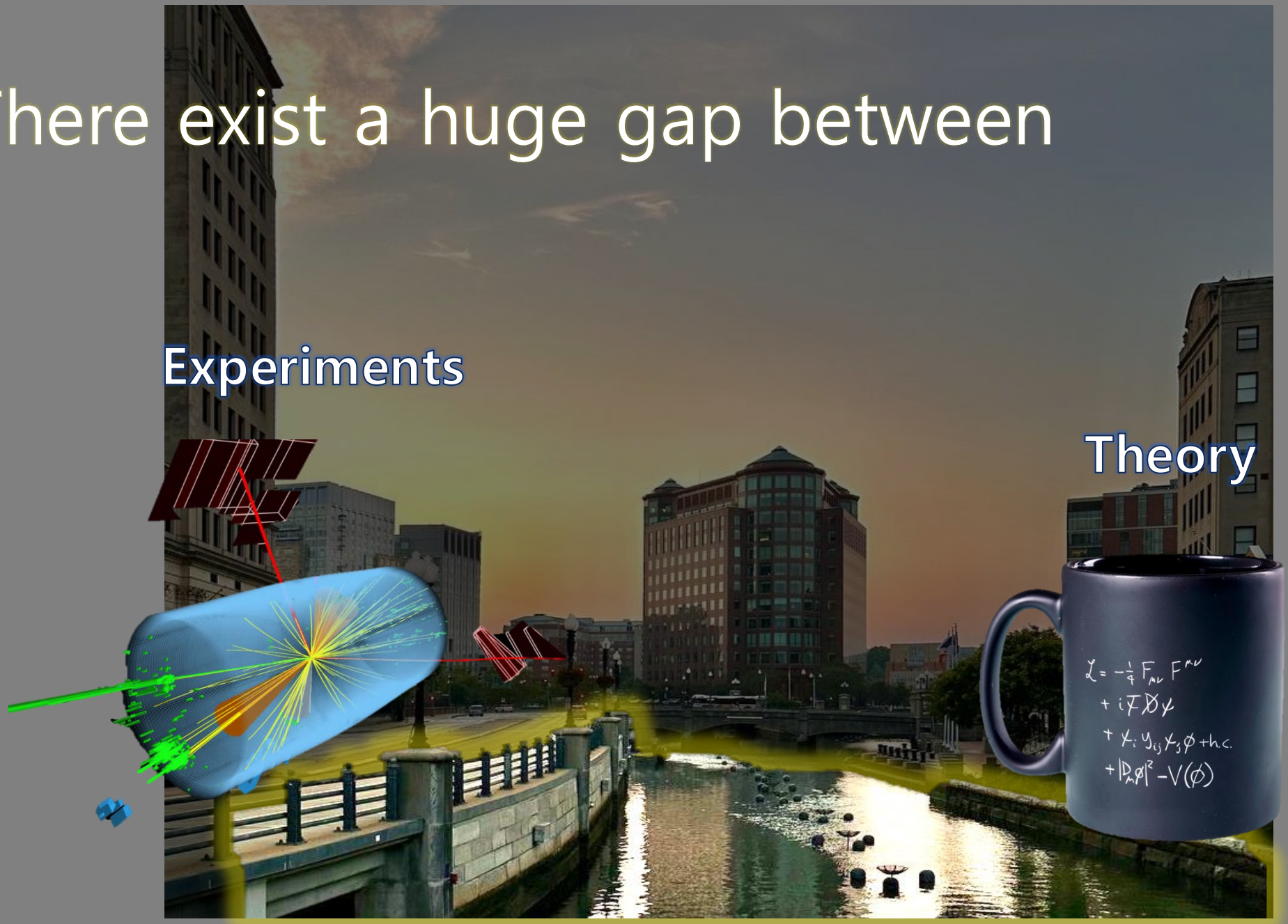
as a part of the MCnet project

funded by Marie Curie Actions

Cropped from **Jennifer's slide** on Monday


this Tuesday
Conference room →
← My hotel

There exist a huge gap between

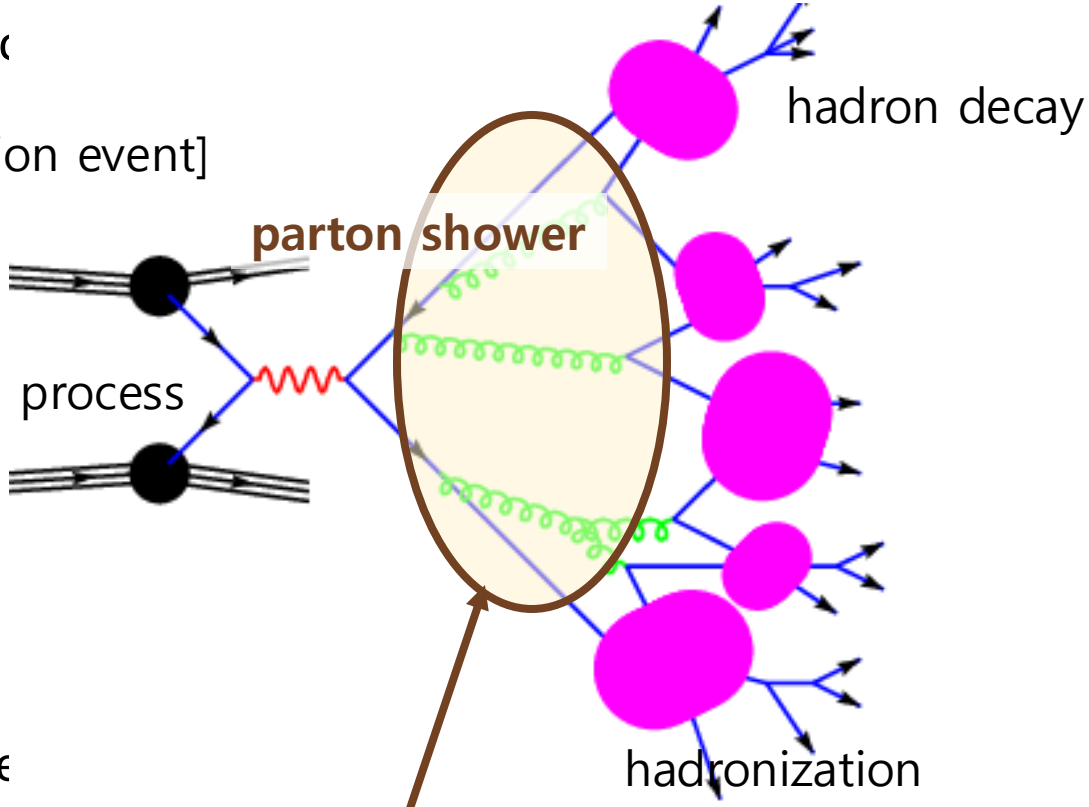


There exist a huge gap between

What are **Event Generators**?

- They translate theoretical models into quantities directly measurable by detectors
  - **dictate real events** as close as pc

[proton-proton collision event]



(Parton Distribution Function)

- Generators simulate all these ste
  Monte Carlo (MC) techniques

This presentation concentrates on this part (**parton shower**)!

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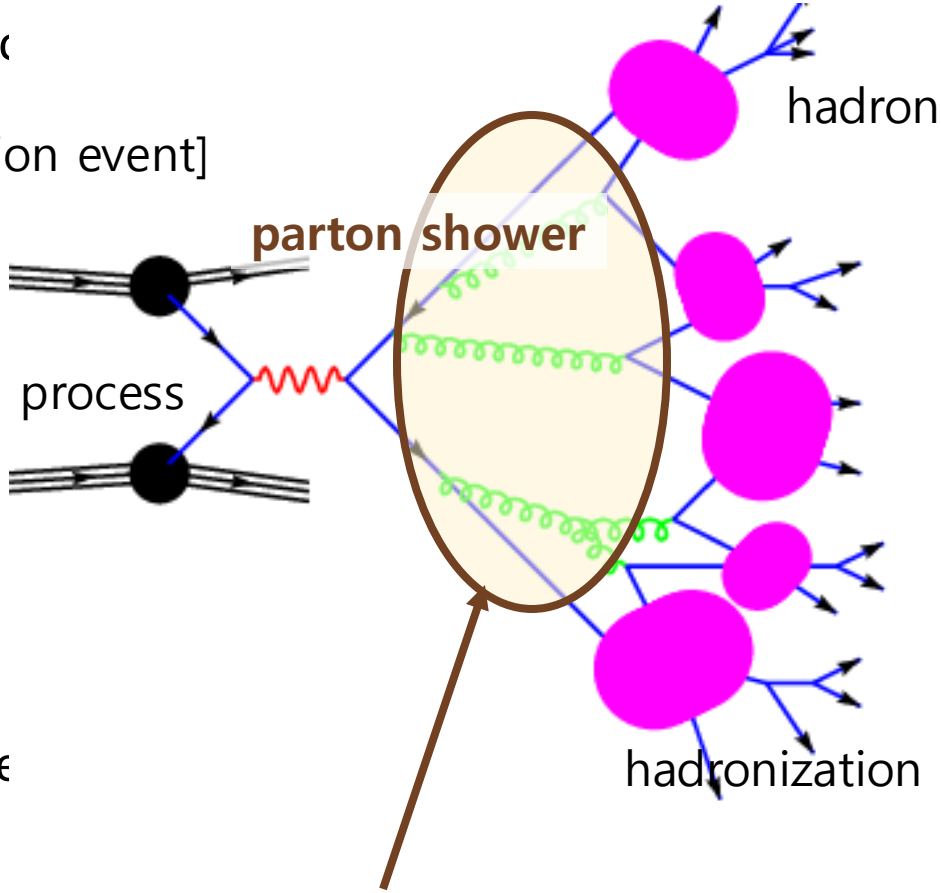

(Parton Distribution Function)

- Generators simulate all these ste
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This presentation concentrates on this part (**parton shower**)!

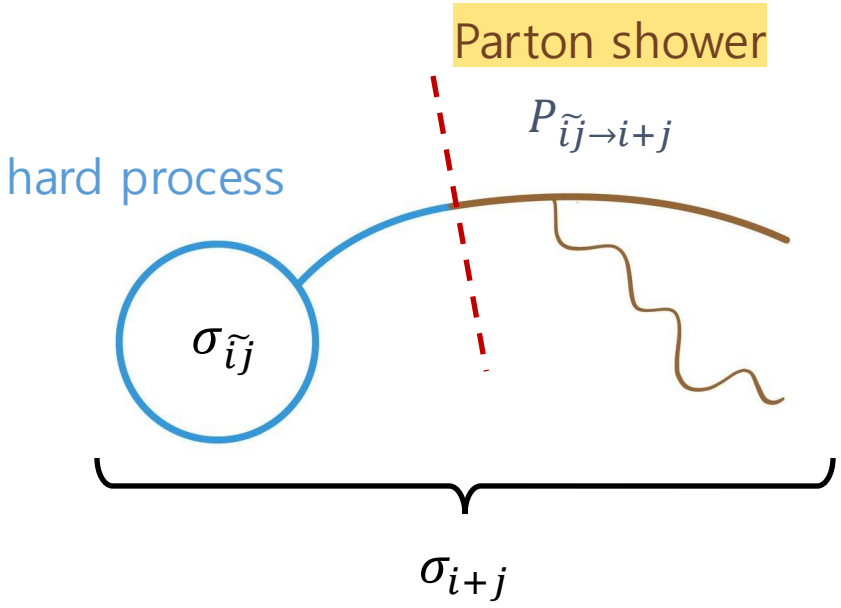
Most important things in this talk are highlighted with:
BRWON and **RED**

BROWN

Parton Shower Algorithm?

- Every coloured particles do QCD showers to make colour singlet final states
- **Each parton radiation** can be treated **separately from** the **hard process**

➡ *" Factorization Theorem "*



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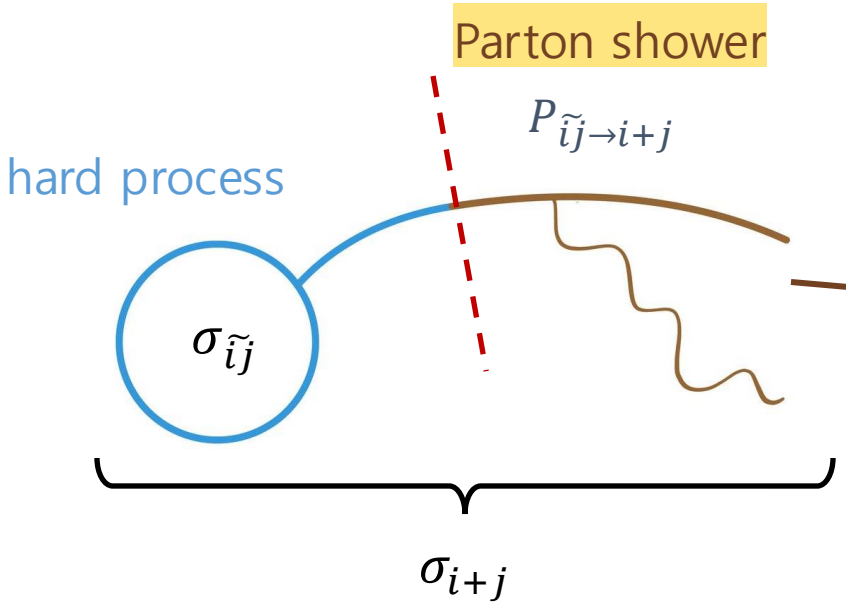


- total cross section

$$d\sigma_{i+j} \simeq \frac{\alpha_{\rm int}(\tilde{q}^2)}{2\pi}\frac{d\tilde{q}^2}{\tilde{q}^2}dz\, P_{\tilde{ij}\to i+j}(z,\tilde{q}) \times d\sigma_{\tilde{ij}}$$

hard process

splitting function

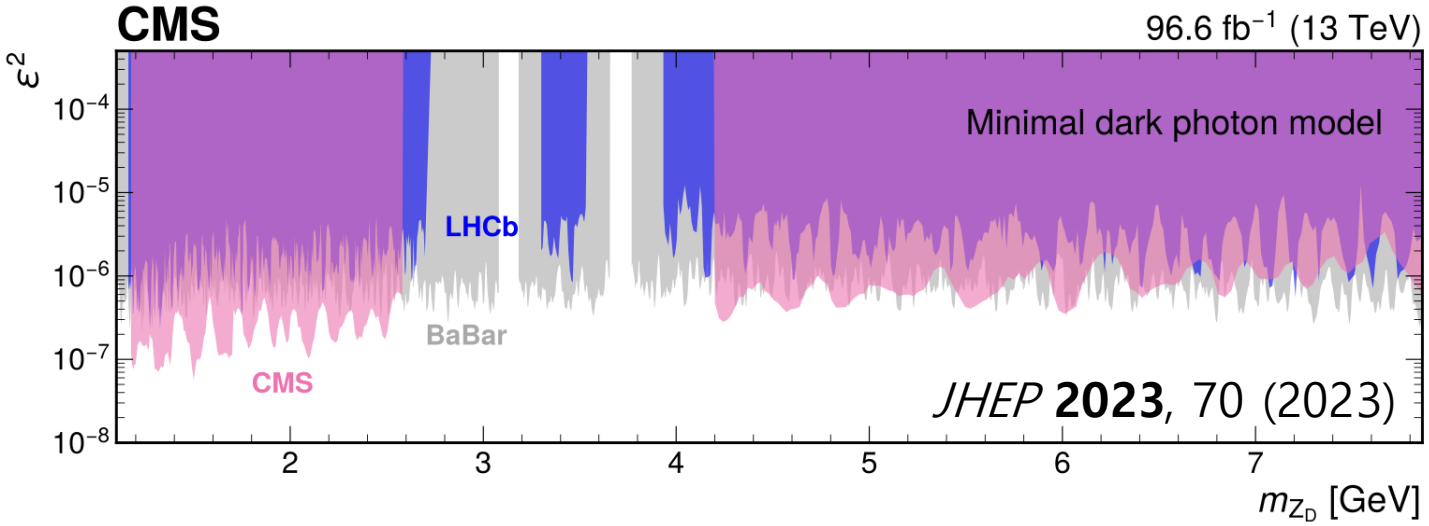
→ The splitting function contains kinematic properties of radiations

z : energy fraction ~ E_{in}/E_{out}
$\tilde{q}$: angular-order variable ~ $\theta \cdot E$

BSM Radiation

MOTIVATION

- **Low scale BSM searches** in LHC down to 1 GeV



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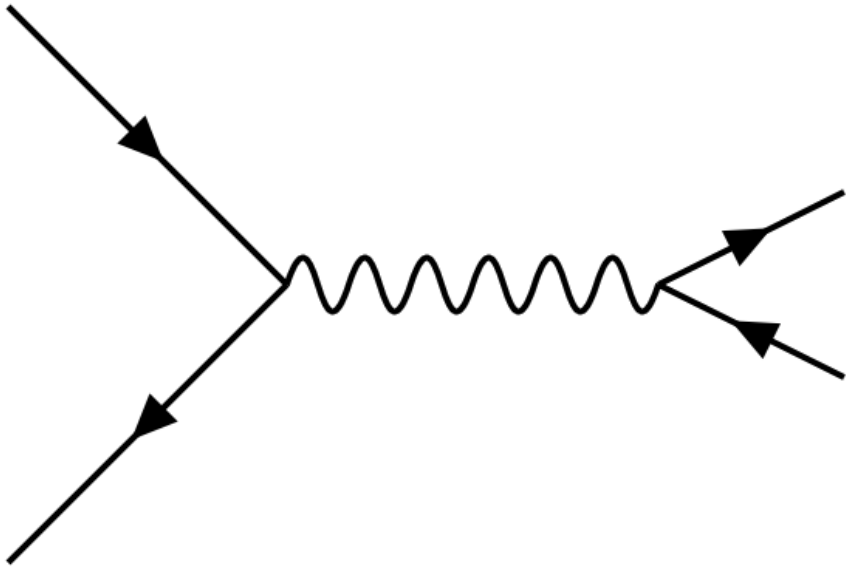

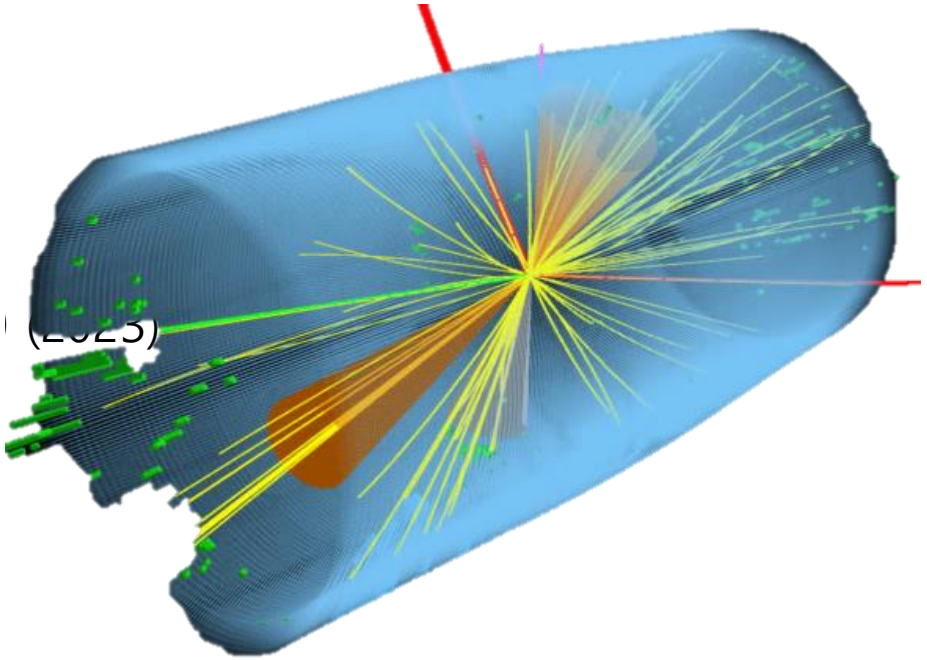

Cannot pass trigger

MOTIVATION

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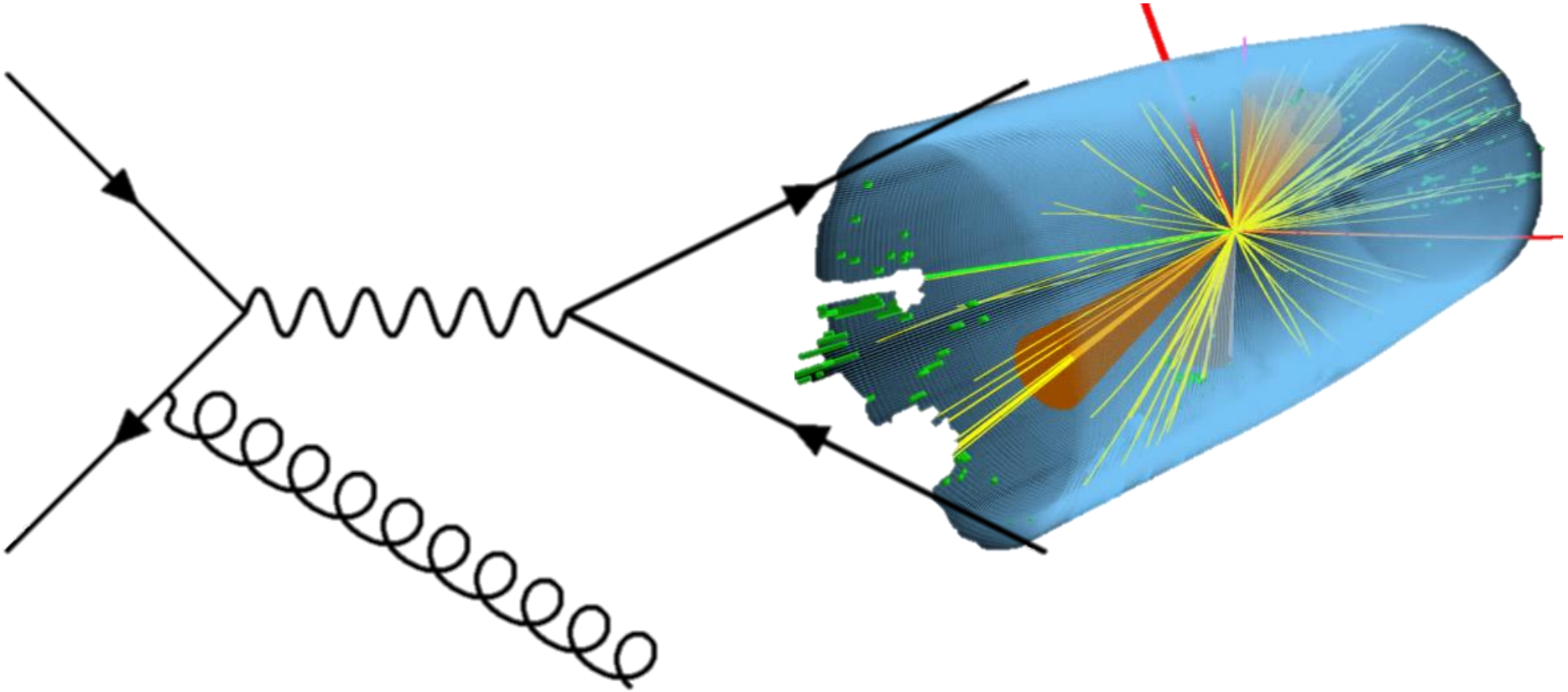
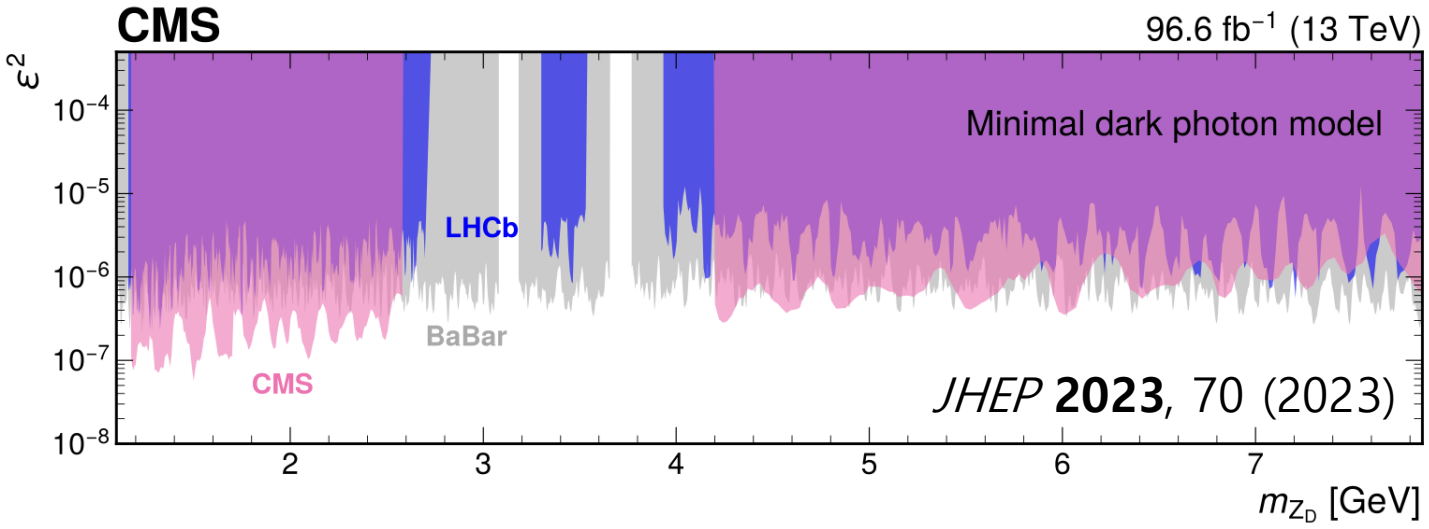


Interplays with jets are neccessitated

MOTIVATION

- **Low scale BSM searches** in LHC



- **Low mass BSM bosons can be produced during the PS** process
  - tones of samples are necessitated over huge parameter spaces
  - *hard to consider all higher order diagrams*
- **The additional time taken by adding BSM radiation grows slowly with the number of emitted particles**

Cropped from **Jennifer's slide** on Monday

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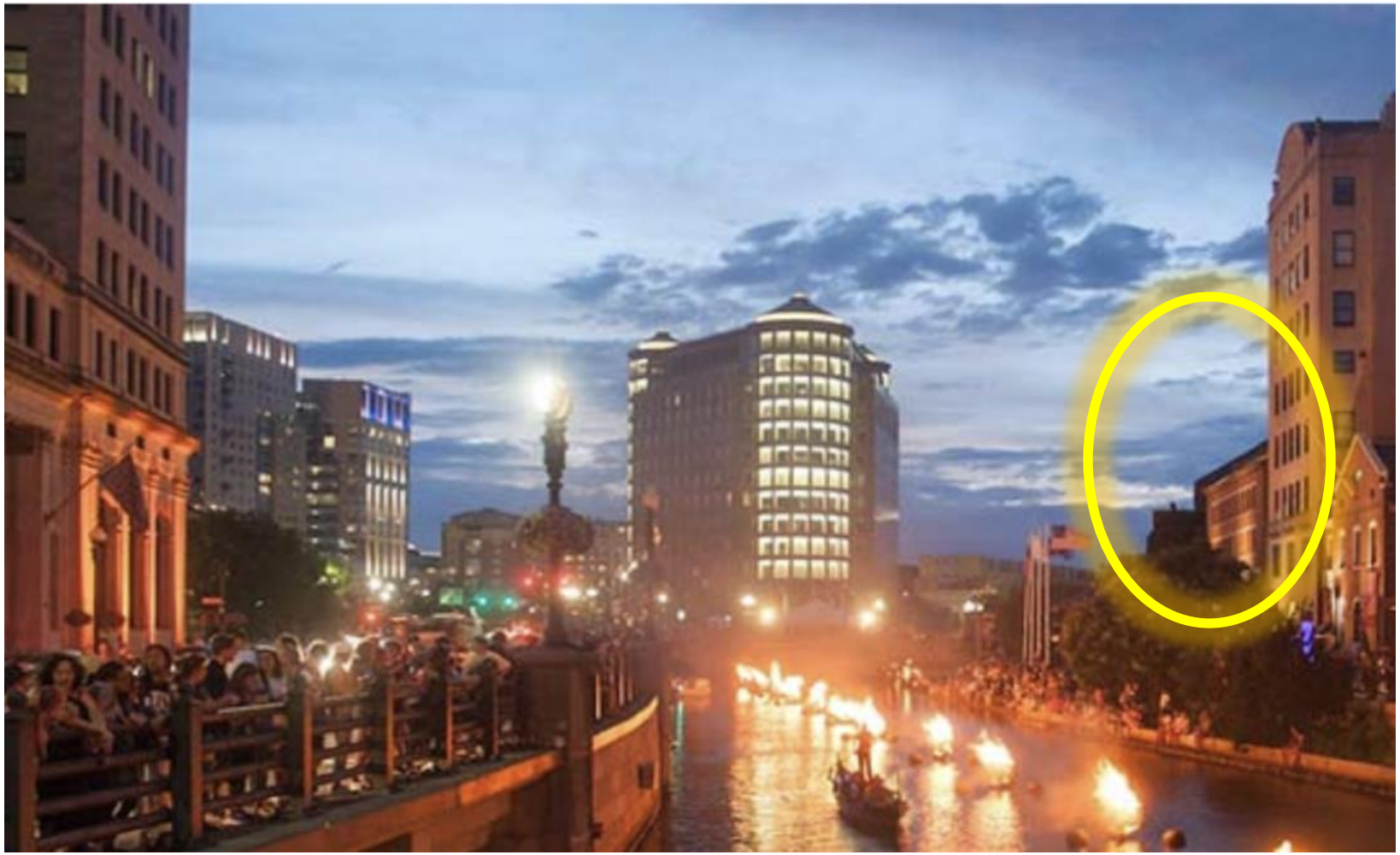
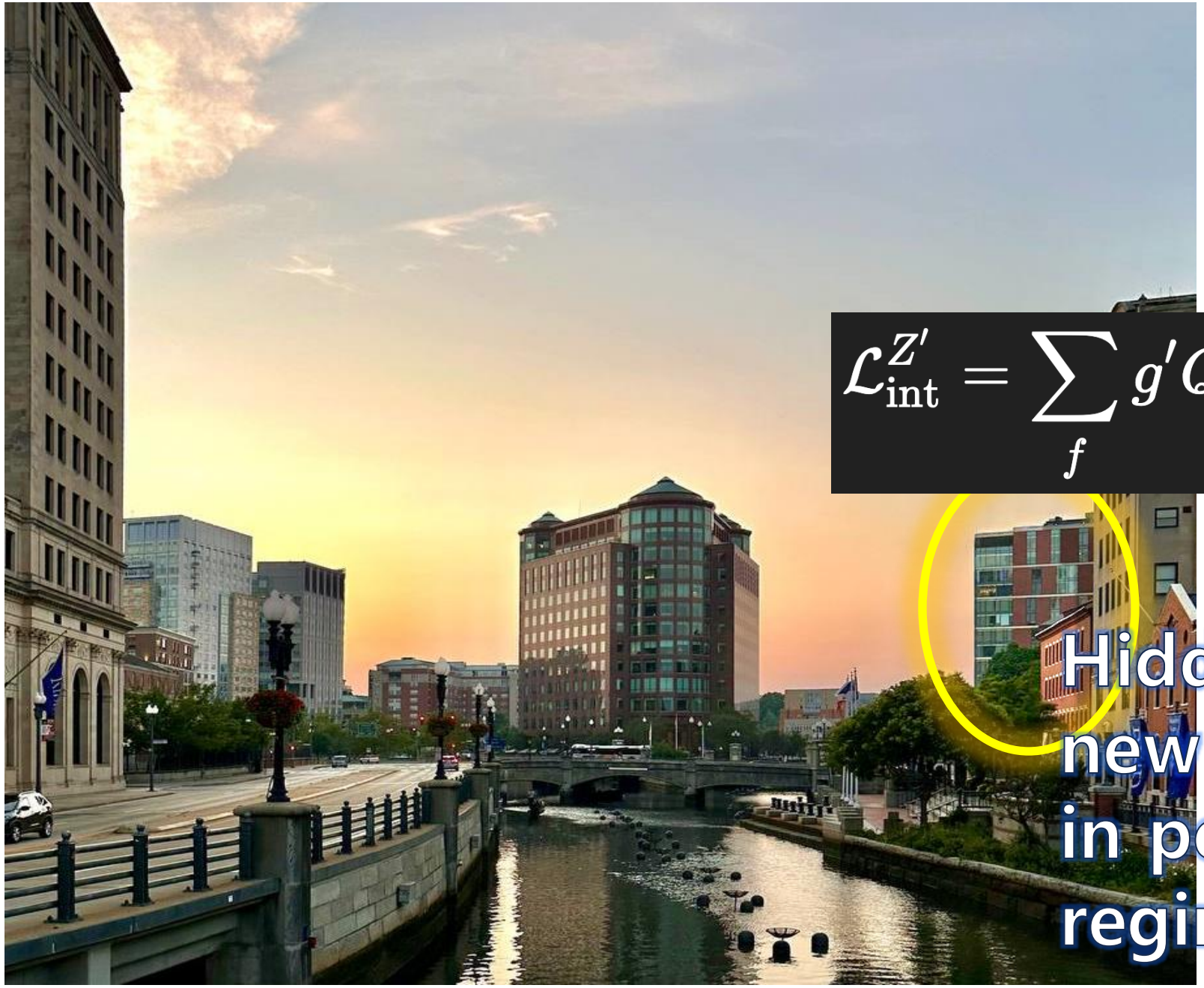


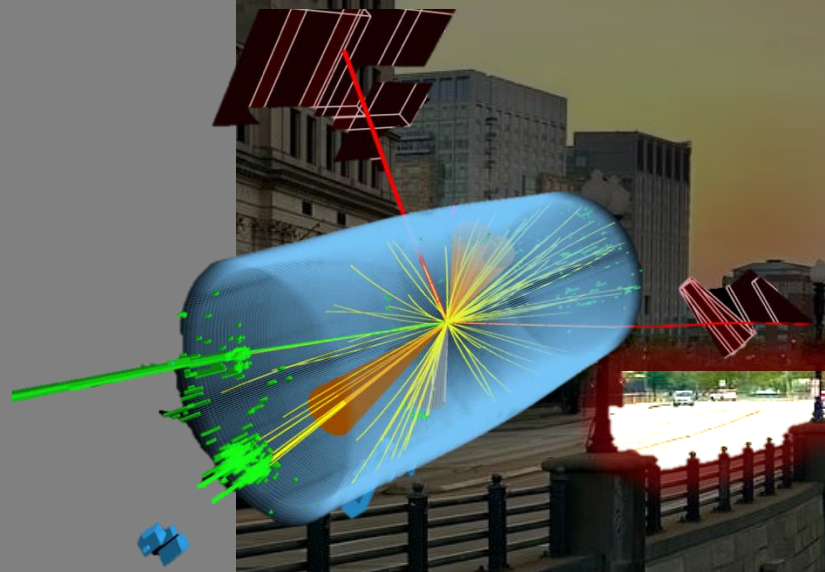


$$\mathcal{L}_{\text{int}}^{Z'} = \sum_f g' Q'_f \bar{f} \gamma^\mu f \, Z'_\mu$$

Hidden new physics in peturbative regime?

Need a **new bridge** for BSM!

Experiments



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Event Generator

Hidden new physics in peturbative regime?

Parton Shower Algorithm?

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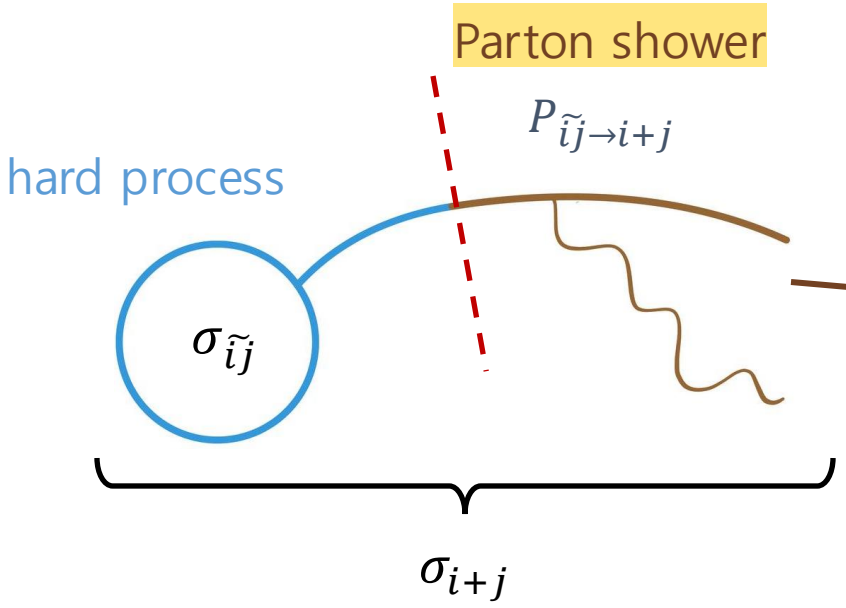

- total cross section

$$d\sigma_{i+j} \simeq \frac{\alpha_{\rm int}(\tilde{q}^2)}{2\pi}\frac{d\tilde{q}^2}{\tilde{q}^2}dz\, P_{\tilde{ij}\to i+j}(z,\tilde{q}) \times d\sigma_{\tilde{ij}}$$

hard process

splitting function

→ The splitting function contains kinematic properties of radiations

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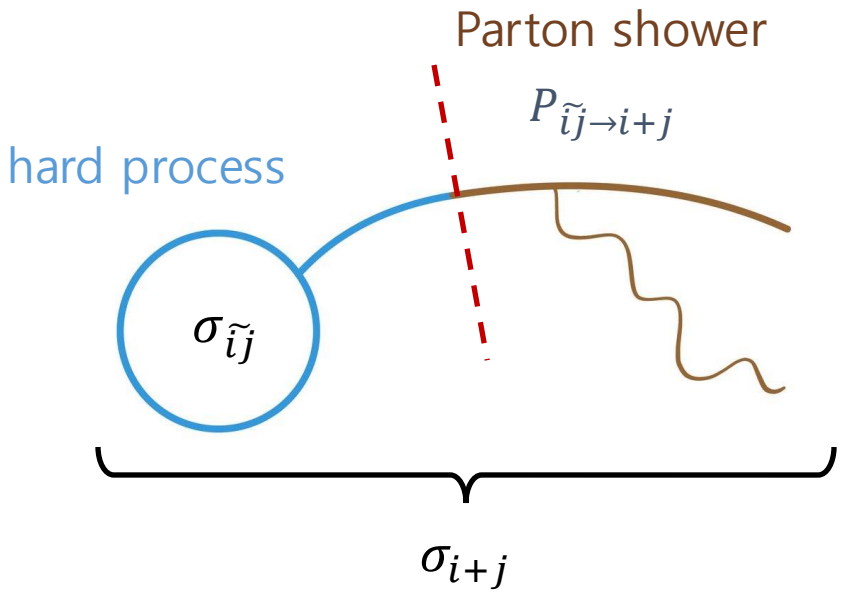
How to?

- Goal: Implement BSM showers as general as possible in Herwig7



- Calculate **splitting functions** for BSM particles

master formula by *Altarelli* and *Parisi*
only **depends on their spin and mass**

$$P_{0\to 12}(z,\tilde{q}) = \frac{1}{2(q_0^2 - m_0^2)} \sum_{s_0,s_1,s_2} |\mathcal{M}_{s_0,s_1,s_2}|^2$$

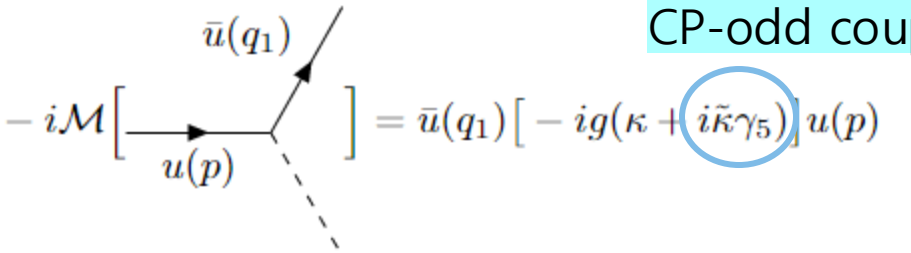
(NPB 126, 2 (1977) 298-318)

for all types of interactions: $\phi \to \phi'\phi''$, $f \to f'\phi$, $V \to V'\phi$
$\phi \to \phi' V$, $f \to f' V$ $V \to V'V''$

See Andrea's slide on Tuesday for massive particle radiations (quasi-collinear limit)

$f \to f' \varphi$ splitting

- Matrix element



$$-i\mathcal{M}\Big[\quad\Big] = \bar{u}(q_1)\big[-ig(\kappa + i\tilde{\kappa}\gamma_5)\big]u(p)$$

CP-odd coupling can be introduced in BSM scenarios

- Spin-dependent **matrix elements**

$\mathcal{M}^{f\to f'\phi}_{\lambda_0,\ \lambda_1}$	$\lambda_1 = \uparrow$	$\downarrow$
$\lambda_0 = \uparrow$	$\frac{\kappa(zm_0+m_1)-i\tilde{\kappa}(zm_0-m_1)}{\sqrt{z}}$	$-\frac{(\kappa+i\tilde{\kappa})p_T \exp i\phi}{\sqrt{z}}$
$\downarrow$	$\frac{(\kappa-i\tilde{\kappa})p_T \exp -i\phi}{\sqrt{z}}$	$\frac{\kappa(zm_0+m_1)+i\tilde{\kappa}(zm_0-m_1)}{\sqrt{z}}$

- Spin-dependent **splitting function**

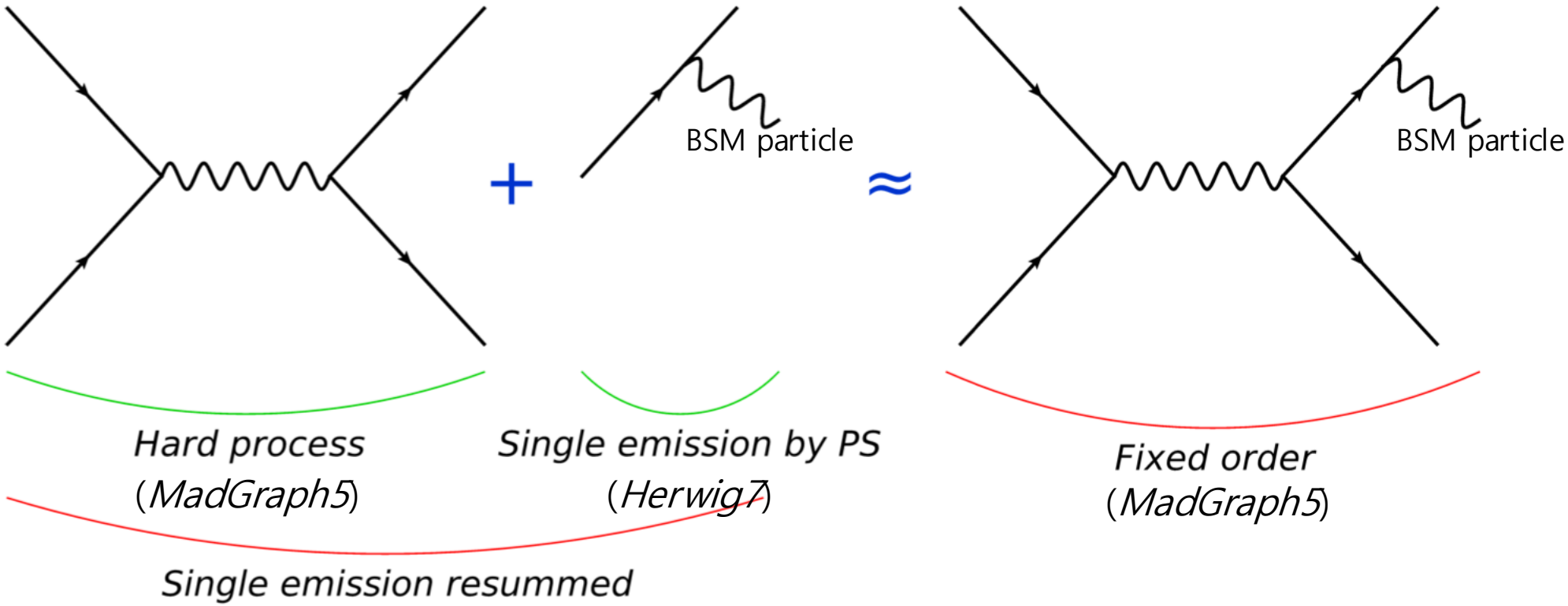
$$P_{f\to f'\phi}(z,\ \tilde{q}) = \frac{g^2}{2}\Big[\left(\rho_+|\kappa+i\tilde{\kappa}|^2+\rho_-|\kappa-i\tilde{\kappa}|^2\right)\cdot\left[(1-z)-m_{2,t}^2\right]+(\rho_++\rho_-)\left[|\kappa|^2(m_{0,t}+m_{1,t})^2 + |\tilde{\kappa}|^2(m_{0,t}-m_{1,t})^2\right]+2(\rho_+-\rho_-)\Im(\kappa\tilde{\kappa})\left[(1-2z)m_{0,t}^2+m_{1,t}^2\right]\Big],$$

→ This explains *both SM higgs* radation & *new scalar particle radiation* out of heavy flavour fermions

Performance test

Resummed shower (RS)
– additional leg from *Herwig7 parton shower*

Fixed-order (FO)
– additional leg from *MadGraph5*

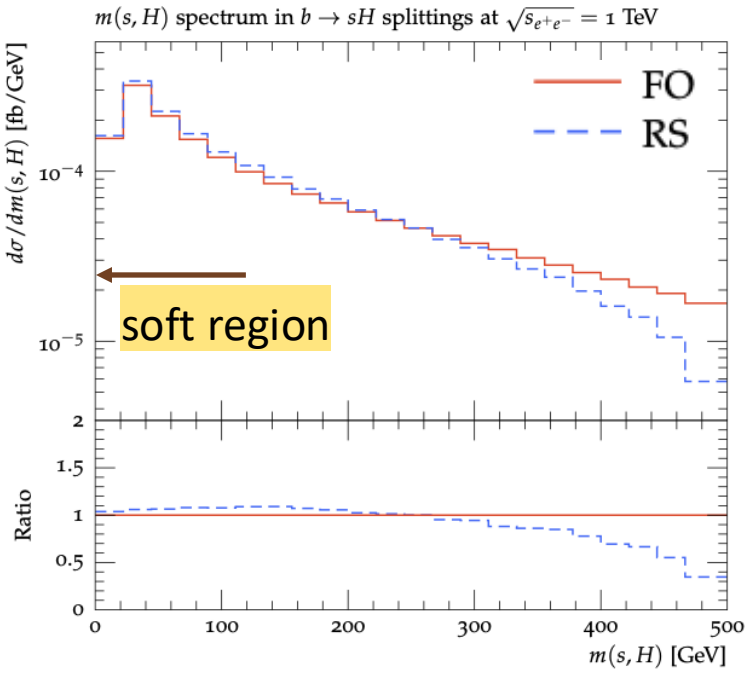


$f \to f' \varphi$ splitting

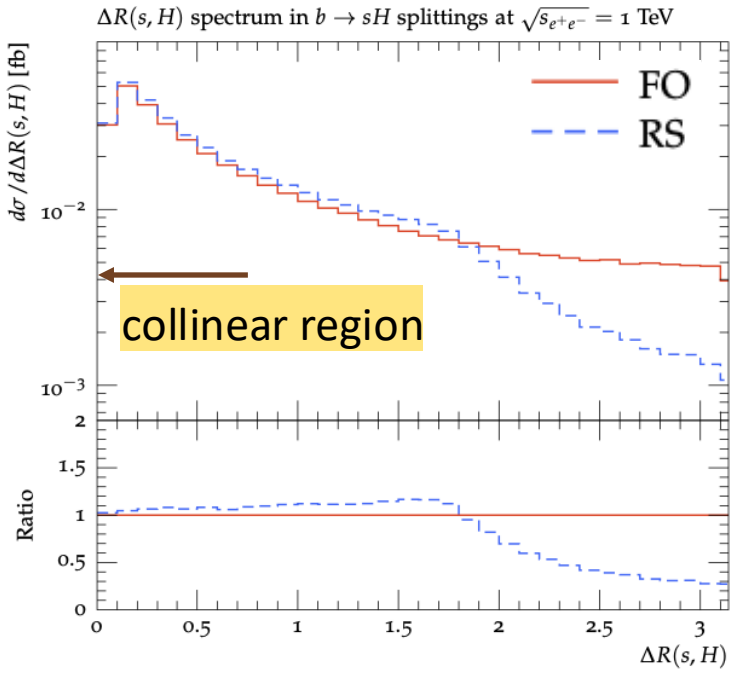
- Important BSM phenomena: **FCNC**!
- **b → sH splitting** in general 2HDM
  - **m(H) = 10 GeV**
- $e^+e^- \to b\bar{b}$ as a baseline @ 1 TeV
  - **Resummed-shower (RS)** result:
    $e^+e^- \to b\bar{b}$ (MG5) + **b** $\to s H$ (HW7)
  - **Fixed-order (FO)** result:
    $e^+e^- \to s\bar{b}H$ (MG5) + counter part

✓ Only one BSM H radiation is allowed even without SM shower for validation

m(b, H)

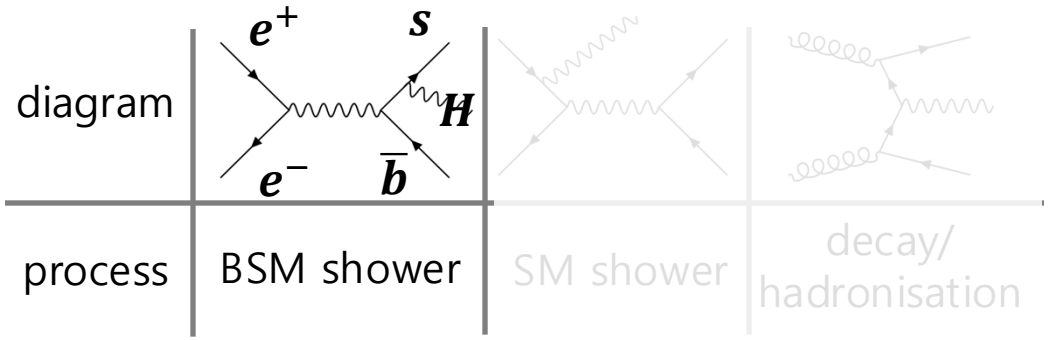


ΔR(b, H)



→ **Wonderful agreements**
especially @ soft & collinear regions

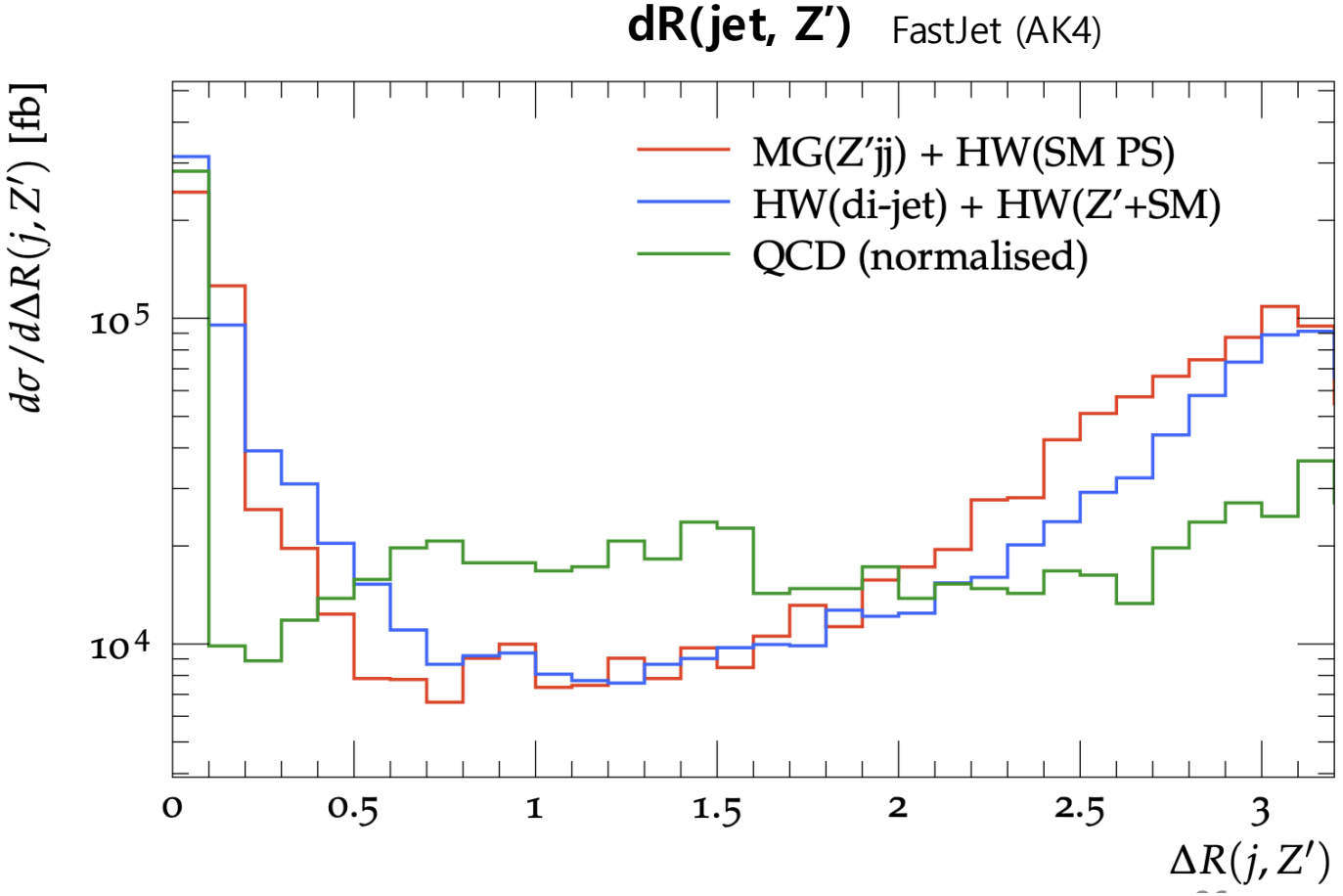
[JHEP 08 (2024) 064]



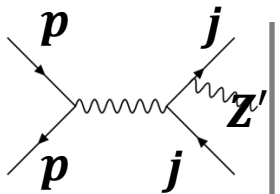
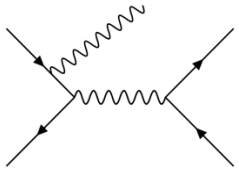
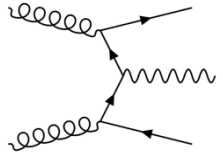
$f \to f' Z'$ splitting

[J.-B. Lee, T. Kim, U.-K. Yang, M.A., M. Seymour]

- Novel chance to study BSM physics in boosted region!
- $p\,p \Rightarrow Z'j\,j$ samples @ $\sqrt{s} = 13$ TeV $(m(Z') = 10$ GeV)



- All included

diagram			
process	BSM shower	SM shower	decay/ hadronisation

How to?

A BSM parton shower in Herwig 7 interface

[JHEP 08 (2024) 064]

Herwig 7 is now equipped to handle all types of BSM boson showers, particularly those without colour structure or additional charges [7–9]. The tutorial section titled "Using UFO Models" provides comprehensive guidance on automatically setting up BSM features using any UFO model file [30]. Preparing Herwig 7 for these simulations involves a few essential steps detailed in this documentation. First, users should allow the BSM shower as follows:

```
ufo2herwig <UFO_directory> --enable-bsm-shower
make
```

will automatically set all BSM splittings

To prevent any kind of disorder caused by including FCNC processes, this command automatically suppresses all FCNC-inducing splittings. One, however, can allow the inclusion of the FCNC processes by adding the "`--allow-fcnc`" flag:

```
ufo2herwig <UFO_directory> --enable-bsm-shower --allow-fcnc
```

Through Herwig 7's `ufo2herwig` module, spin information, inter values, and others related to the parton shower process are written file, which can be read by

```
read FRModel.model
```

Herwig 7.4 public release

now one can download Herwig 7.4α on my private git

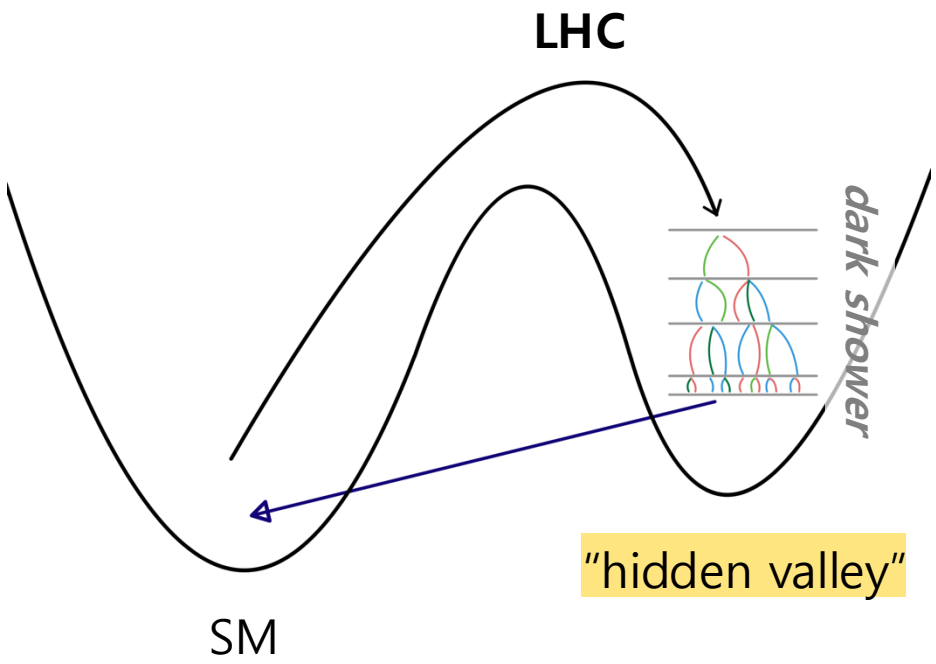
Extensions of BSM parton shower

1. Dark shower in Herwig7
   - ✓ Implementation of the hidden valley model
   - ✓ Splitting kernels are exactly same as before
   - ✓ SU(3) dark interaction tree, dark quarks, dark cluster hadronisation, and decay

   [Eur. Phys. J. C 84 (2024) 1210]

2. *Dark photon* production in Herwig7
   - ✓ Inclusion of light dark portal model
   - ✓ Dark photon radiation & bremsstrahlung

   [Kling, Pläzer, Reimitz, AM]



3. Extension to exotic fermions ($s = 3/2$) and gravitons

Summary

- **BSM showers are implemented in Herwig7**

- Results are validated by comparing with MadGraph5 LO simulations

  → show great agreements @ soft/collinear regions

  → with some well-understood discrepancies

Take home messages

- For low scale BSM searches, BSM + multi-jet processes are necessary

- BSM shower is an efficient way to do this!

BOOSTAMOS

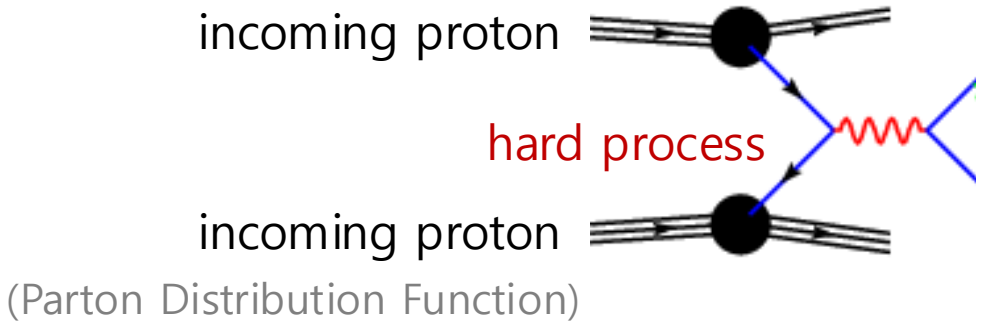
Brief Introduction to Herwig 7

- Event Generators
- Parton Shower Algorithm
- Angular-ordering

What are Event Generators?

- They translate theoretical models into quantities directly measurable by detectors
  - dictate real events as close as possible

[proton-proton collision event]



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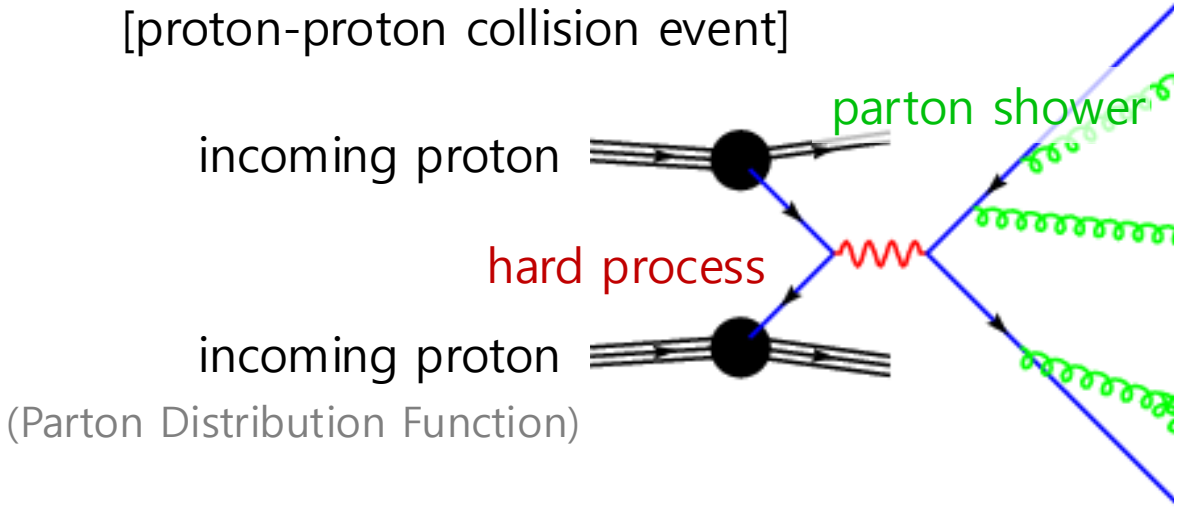

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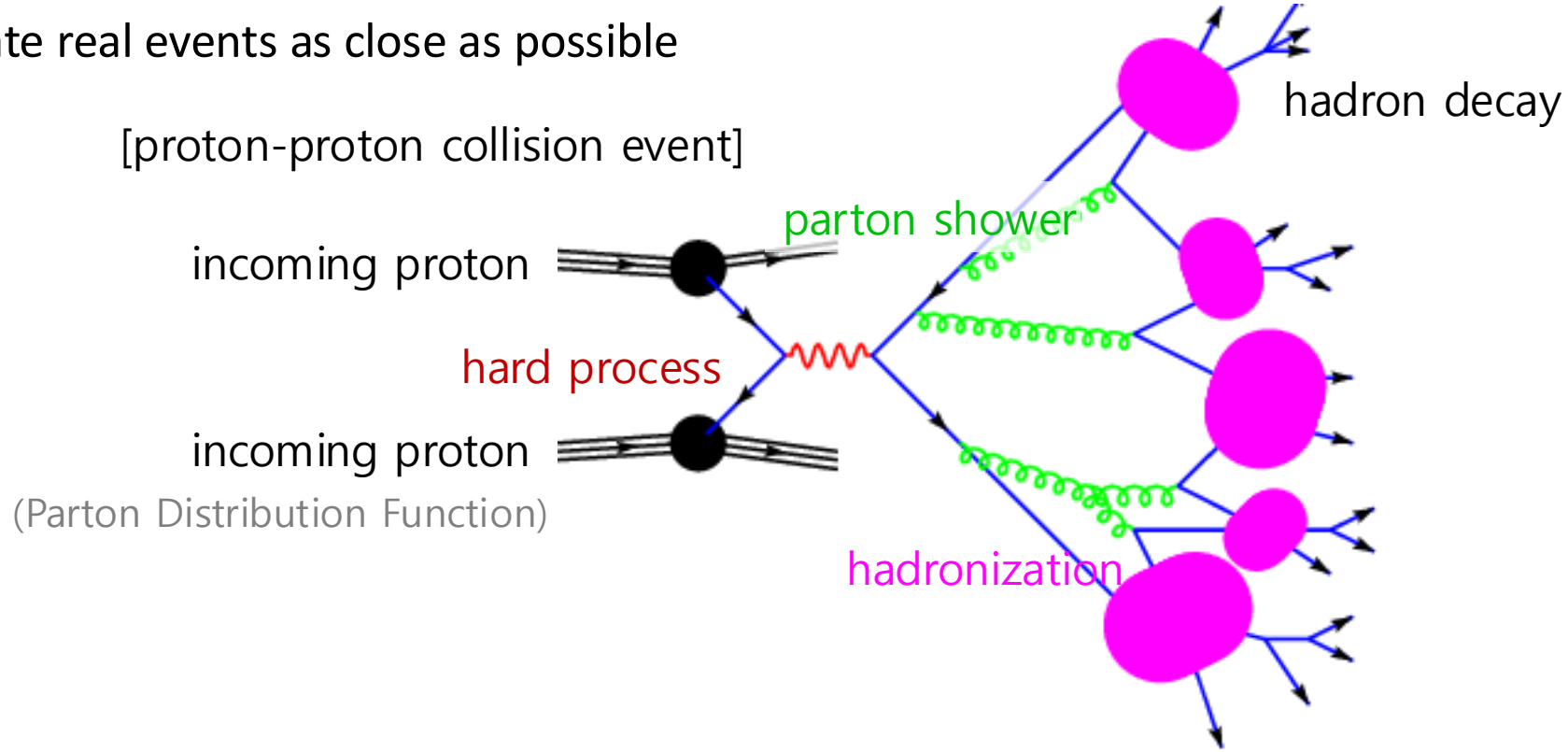
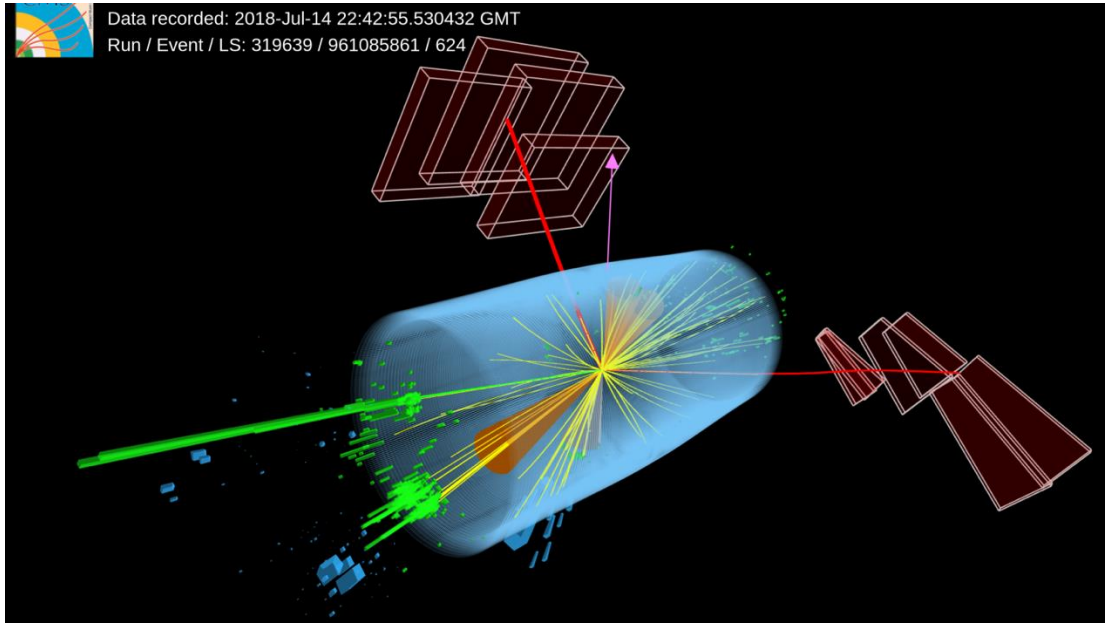


Why Monte Carlo integration?

- **LHC gives ~50 charged tracks** for each event
  → at least 150-dim integration



Why Monte Carlo integration?

- **LHC gives ~50 charged tracks** for each event

  → at least 150-dim integration

- Traditional integration algorithms necessitates regular n^d grids
  - n: integration points, d: dimension of the integration
  - e.g. Trapezoidal Rule

$$\int_a^b f(x)dx \approx \sum_{k=1}^{N} \frac{f(x_{k-1}) + f(x_k)}{2} \Delta x_k$$

- **MC integration**
  - Law of large numbers (LLN) – a huge number of independent and identical random samplings make its average value converge to the true value

  → It makes the **accuracy only depends on the number of random points** independent to the dimension

[Relative uncertainty of each integration method]

Convergence Rate	1-dim	d-dim
Trapezoidal Rule	$1/n^2$	$1/n^{2/d}$
Simpson's Rule	$1/n^4$	$1/n^{4/d}$
Monte Carlo Method	$1/\sqrt{n}$	$1/\sqrt{n}$

traditional methods depending on d

independent to d

Herwig7: angular-order parton shower

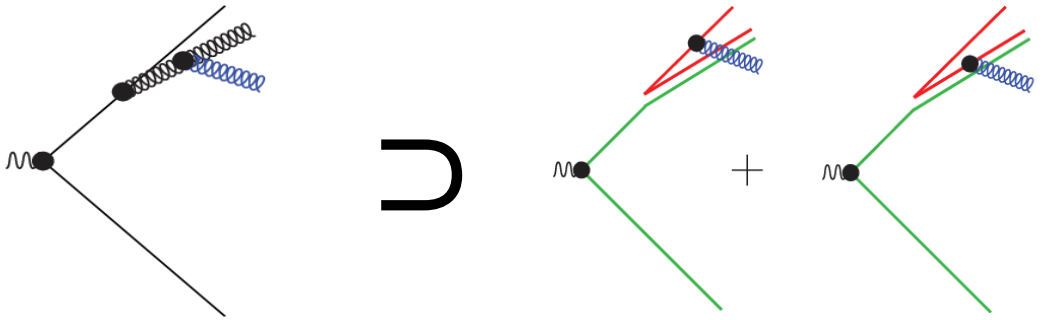
- Shower algorithms

	Herwig7	PYTHIA8
shower algorithm	angular-order	pT-ordered dipole shower

- When we consider parton showers, colour coherence is a great issue

(Most of old generators(PYTHIA6 …) cannot treat this effect correctly)

interference → colour coherence



Herwig7: angular-order parton shower

- Shower algorithms

	Herwig7	PYTHIA8
shower algorithm	angular-order	pT-ordered dipole shower

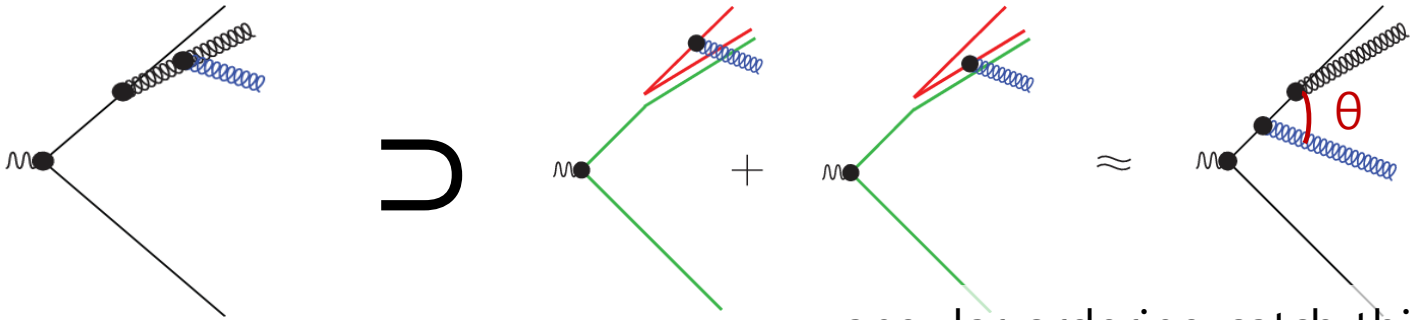
- When we consider parton showers, colour coherence is a great issue

(Most of old generators(PYTHIA6 …) cannot treat this effect correctly)

[Chudakov effect]

A radiating parton (blue gluon) only sees the charge of the initial parton (green) = total charge of the final partons

interference → colour coherence



angular-ordering catch this effect completely!

Parton Shower in Event Generators

1. Find out an evolution variable, q^2, at the point when a radiation occurrs

   Select a random number, $\rho \in [0, 1]$, and solve

   - No emission probability: $\Delta_i(Q^2, q^2) = \rho$

   to find a q^2. Finally, if

   (a) $q^2 > Q_0^2$: New branching is generated.

   (b) $q^2 < Q_0^2$: Evolution is terminated.

2. Fix a branching type

   Normalize the integrated emission probabilities as follows

   - Splitting probability: $\mathcal{P}_{ji} = \frac{\int_{z_{min}}^{z_{max}} P_{ji}(z)dz}{\int_{z_{min}}^{z_{max}} \sum_k P_{ki}(z)dz}$. (2.18)

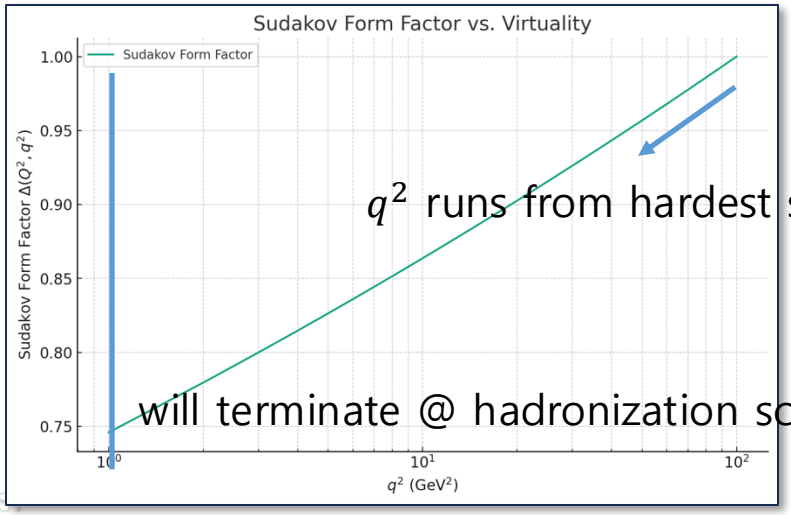
(integrated & normalized splitting function)

   Select a random number, $\lambda \in [0, 1]$.

   (a) If $\lambda < \mathcal{P}(g \to q\bar{q})$, choose $q\bar{q}$ splitting.

   (b) If $\mathcal{P}(g \to q\bar{q}) < \lambda < \mathcal{P}(g \to q\bar{q}) + \mathcal{P}(g \to gg)(= 1$, in this case), choose gg splitting.

[ChatGPT4.0 – Sudakov Form Factor]



q^2 runs from hardest scale, e.g. m_Z^2

will terminate @ hadronization scale (Q_0^2) ~ GeV

Parton Shower in Event Generators

1. Find out an evolution variable, q^2, at the point when a radiation occurrs

   Select a random number, $\rho \in [0, 1]$, and solve

   - No emission probability: $\Delta_i(Q^2, q^2) = \rho$ (2.17)

   to find a q^2. Finally, if

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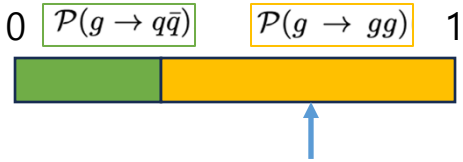
(integrated & normalized splitting function)

- normalized splitting probability:



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   (b) If $\mathcal{P}(g \to q\bar{q}) < \lambda < \mathcal{P}(g \to q\bar{q}) + \mathcal{P}(g \to gg)(= 1$, in this case), choose gg splitting.

Parton Shower in Event Generators

3. Find out z

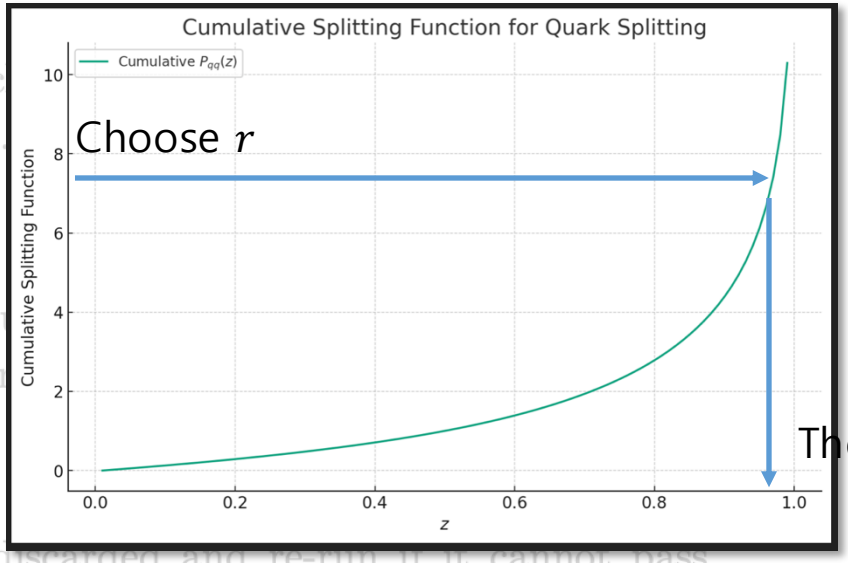
Again choose a random number, $r \in [0, 1]$. Solve the following equation:

$$r \int_{z_{min}}^{z_{max}} P(z)dz = \int_{z_{min}}^{z} P(z')dz', \qquad (2.19)$$

where $P(z)$ is a splitting function, showing a likelihood of the variable, z.

* Rejection sampling

[ChatGPT4.0 - cumulative q→qg splitting function]



Then it gives z

4. Fix kinematics

Choose all the other dependent variables, suc
q^2 and z, where ϕ is almost randomly chosen.

5. Kinematic mapping

All variables have been calculated in the S
shower coordinates. Apply boostings, rotatio

6. Apply phase space constraints

To avoid the divergence issues, events are discarded and re-run if it cannot pass
virtuality or p_T cutoffs.

Parton Shower in Event Generators

3. Find out z

   Again choose a random number, $r \in [0, 1]$. Solve the following equation:

$$r \int_{z_{min}}^{z_{max}} P(z)dz = \int_{z_{min}}^{z} P(z')dz', \tag{2.19}$$

   where $P(z)$ is a splitting function, showing a likelihood of the variable, z.

   * Rejection sampling

4. Fix kinematics

   Choose all the other dependent variables, such as p_T, ϕ, and 4-momentum based on q^2 and z, where ϕ is almost randomly chosen.

5. Kinematic mapping

   All variables have been calculated in the Sudakov basis or corresponding parton shower coordinates. Apply boostings, rotations and so on to make the lab frame.

6. Apply phase space constraints

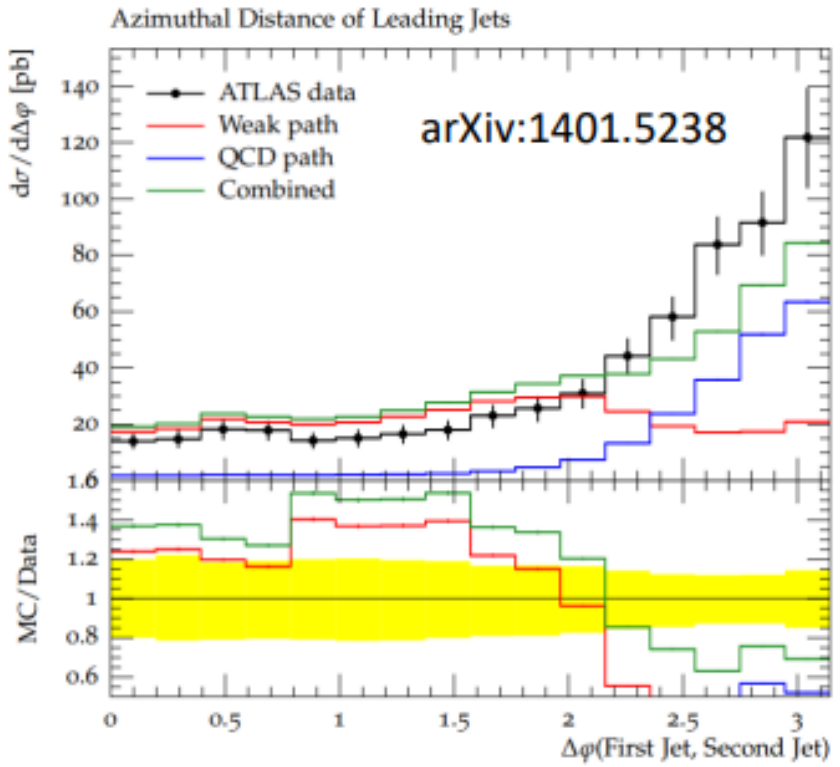
   To avoid the divergence issues, events are discarded and re-run if it cannot pass virtuality or p_T cutoffs.

BSM Parton Shower Algorithm

- Motivation
- Splitting Functions
- Validations

Motivations

- Radiation of a BSM particle can distort kinematic distributions



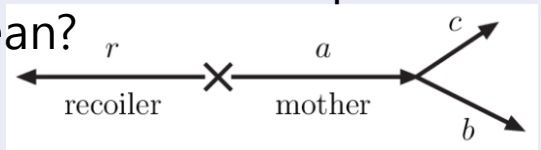
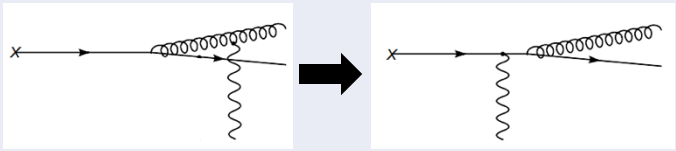
← Weak boson radiation example

Δϕ distribution between leading two jets in ATLAS W+jets sample

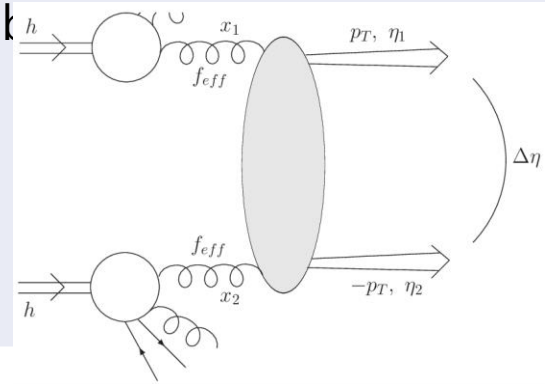
- Weak path: W sample + QCD radiation
- QCD path: QCD sample + QCD/weak radiation

! QCD path changes the distribution shape !
! Same thing can happen for the BSM radiation !

PYTHIA vs HERWIG

	PYTHIA	HERWIG
shower algorithm	pT ordered dipole shower	angular order (averaging a radiation along the radiator)
	What does the "dipole" mean?  E-p conservation is achieved by balancing the momentum of the daughter partons and it of recoiler	 "Chudakov effect" A radiating parton only sees the total charge of initial partons
accuracy	only LL accuracy	+ NLL (+ NNLL, process dependent)
limitation		cannot be used for some ob / jets

Is it right?



Herwig7

- color coherence -> angular order
- splitting function -> intuitive
- no ambiguity in a parton history

$$\frac{\mathrm{d}\theta^2}{\theta^2} = \frac{\mathrm{d}q^2}{q^2} = \frac{\mathrm{d}k_\perp^2}{k_\perp^2}$$

give identical results in the collinear limit, but different extrapol
ations away from it

INTRODUCTION

- Goal: Implement BSM showers as general as possible in Herwig7

- Step by step guidelines
  1) Calculate matrix elements of each process from Feynman rules
     → only depends on their spin and mass
  2) Calculate new splitting functions (NPB 126, 2 (1977) 298-318)
     - master formula by Altarelli and Parisi

$$P_{0\to 12}(z,\tilde{q}) = \frac{1}{2(q_0^2 - m_0^2)} \sum_{s_0,s_1,s_2} |\mathcal{M}_{s_0,s_1,s_2}|^2$$

  3) Herwig7 reads BSM model information and calculate all necessary ingredients automatically

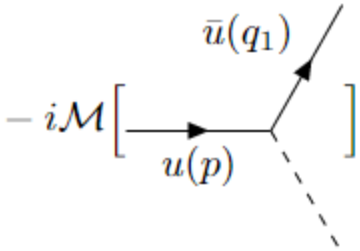
model independent

No needs to calculate shower process for each BSM model/particle

- Important processes
  1. Splitting out of scalar bosons: $\phi \to \phi'\phi''$, $f \to f'\phi$, $V \to V'\phi$
  2. Splitting out of vector bosons: $\phi \to \phi' V$, $f \to f' V$, $V \to V'V''$
  3. All the other combinations can be calculated by symmetries

(spin) 1/2 → 1/2 0 splitting

- Matrix element of q→qφ



$$-i\mathcal{M}\Big[\quad\Big] = \bar{u}(q_1)\big[-ig(\kappa + i\tilde{\kappa}\gamma_5)\big]u(p)$$

CP-odd coupling is added

- Spin-dependent matrix elements

$\mathcal{M}^{f\to f'\phi}_{\lambda_0,\ \lambda_1}$	$\lambda_1 = \uparrow$	$\downarrow$
$\lambda_0 = \uparrow$	$\frac{\kappa(zm_0+m_1)-i\tilde{\kappa}(zm_0-m_1)}{\sqrt{z}}$	$-\frac{(\kappa+i\tilde{\kappa})p_T \exp i\phi}{\sqrt{z}}$
$\downarrow$	$\frac{(\kappa-i\tilde{\kappa})p_T \exp -i\phi}{\sqrt{z}}$	$\frac{\kappa(zm_0+m_1)+i\tilde{\kappa}(zm_0-m_1)}{\sqrt{z}}$

- Spin-dependent splitting function

"generalized" splitting function

$$P_{f\to f'\phi}(z,\ \tilde{q}) = \frac{g^2}{2}\Big[\left(\rho_+|\kappa+i\tilde{\kappa}|^2+\rho_-|\kappa-i\tilde{\kappa}|^2\right)\cdot\left[(1-z)-m_{2,t}^2\right]+(\rho_++\rho_-)\Big[|\kappa|^2(m_{0,t}+m_{1,t})^2$$
$$+|\tilde{\kappa}|^2(m_{0,t}-m_{1,t})^2\Big]+2(\rho_+-\rho_-)\Im(\kappa\tilde{\kappa})\left[(1-2z)m_{0,t}^2+m_{1,t}^2\right]\Big],$$

- SM q→qH splitting function

$$P^{SM}_{f\to f\phi}(z,\tilde{q}) = \frac{g_W^2}{8}\left(\frac{m_0}{m_W}\right)^2\left[(1-z)+\frac{4m_0^2-m_2^2}{z(1-z)\tilde{q}^2}\right].$$

← SM setup

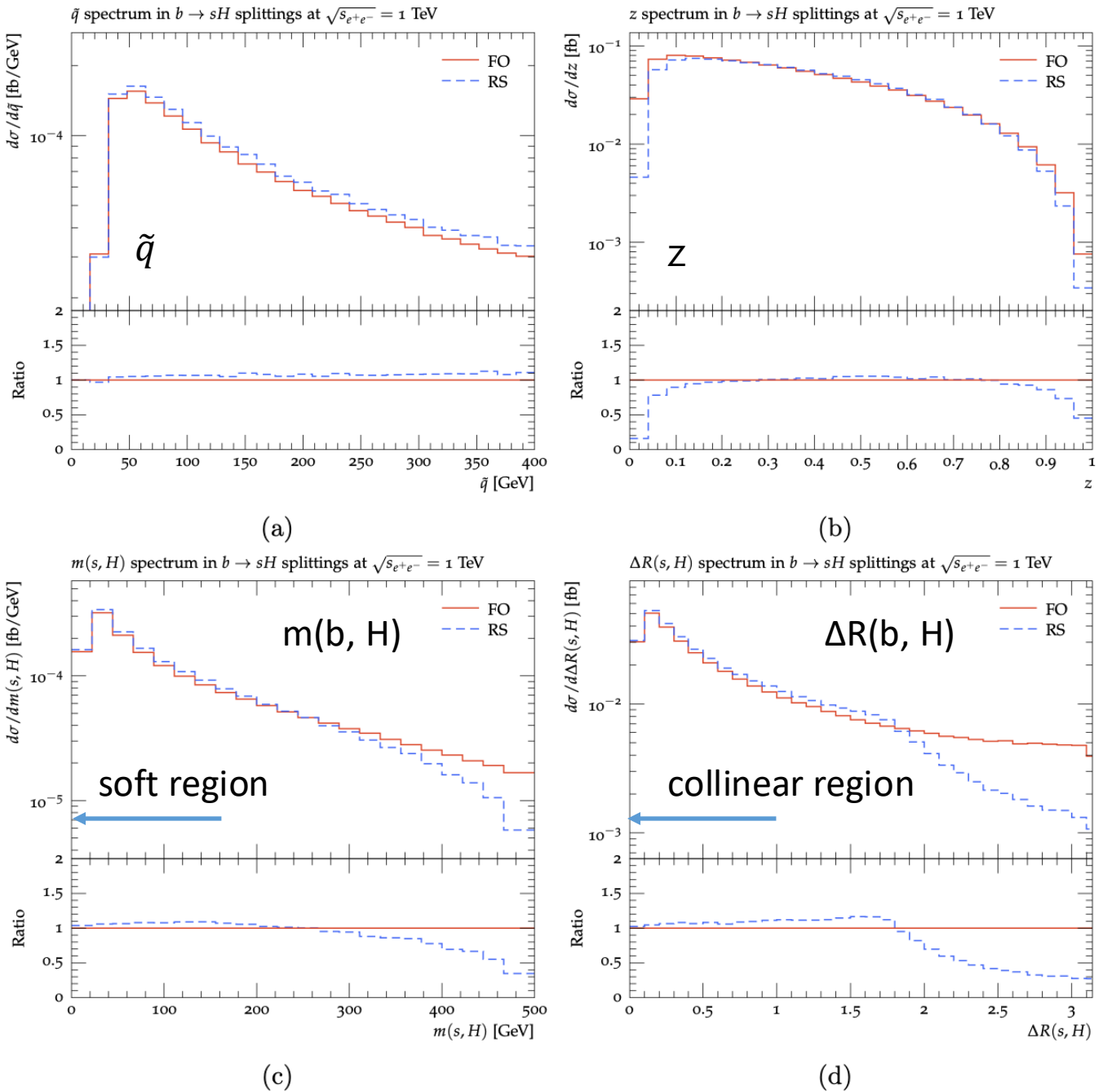
$$g = g_W m_0/2m_W,\ g_W = e/\sin\theta_W$$
$$m_0 = m_1,\ \kappa = 1,\ \tilde{\kappa} = 0,\ \text{and}\ \rho_0+\rho_1 = 1,$$

(spin) 1/2 → 1/2 0 splitting

- Important BSM phenomena: **FCNC**!
- **b → sH splitting** in general 2HDM
  - **m(H) = 10 GeV**
- $e^+e^- \to b\bar{b}$ as a baseline @ 1 TeV
  - **Resummed-shower (RS)** result:
    $\boldsymbol{e^+e^- \to b\bar{b}}$ (MG5) **+ b →** $\boldsymbol{sH}$ (HW7)
  - **Fixed-order (FO)** result:
    $\boldsymbol{e^+e^- \to s\bar{b}H}$ (MG5) + counter part
    (All irrelevant diagrams are removed)

✓ Only one BSM H radiation is performed even without SM shower for validation

*** Wonderful overall agreements**



How to

- PS concept

- What we should do when developing new PS processes
  - = add new splitting functions

How to calculate

- 1->2 splitting
- angular order
- spin-independent – spin matrix of an incoming particle

- Why matrix element for 1->2? (+citation)

Parton Showers

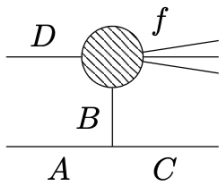
- perturbative
- factorization theorem says
  - the cross section for the emission of an extra parton is fully factorised

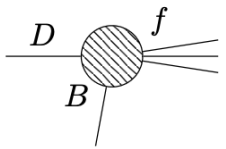
https://arxiv.org/pdf/1904.11866.pdf

- (n+1)/n
- sudakov form fact
- splitting function

$$\mathcal{S} = \frac{d\sigma_{q\bar{q}g}}{\sigma_{q\bar{q}}} = \sum_{i=\text{partons}} \frac{\alpha_s}{2\pi} \frac{dq^2}{q^2} dz P_{ji}(z,q)$$

Splitting function





$$-i\mathcal{M}_{A+B\to C+f} = g^2\frac{(-iV_{A\to B+C})(-iV_{B+D\to f})}{q^2} \qquad -i\mathcal{M}_{B+D\to f} = g(-iV_{B+D\to f})$$

$$d\sigma_{A+B\to C+f} = d\mathcal{P}_{A\to BC}(z)dzd\sigma_{B+D\to f}$$

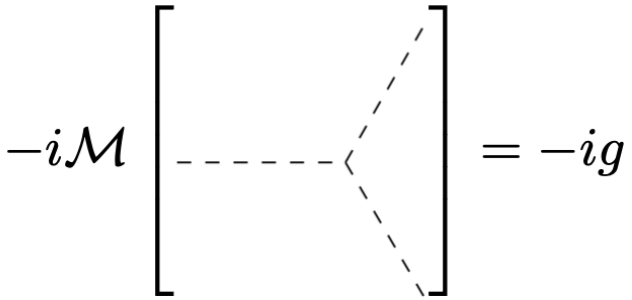
$$P_{A\to BC} = \frac{z(1-z)}{2}\sum_{spins}\frac{|V_{A\to BC}|^2}{p_T^2}$$

Sudakov form factor

Simplest case: (spin) 0 → 00 splitting

- Fully symmetric and no spins!

  → gives constant matrix element



→ No parton indices occured

- Splitting function from the master formula

$$P_{\phi\to\phi'\phi''}(z,\tilde{q}) = \frac{g^2}{2Sz(1-z)\tilde{q}^2}$$

- symmetric factor

$S = 1$ for $\phi' \neq \phi''$ and $S = 2$ for $\phi' = \phi''$

Simplest case: (spin) 0 → 00 splitting

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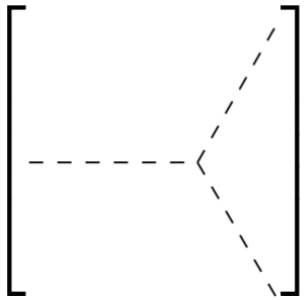
$-i\mathcal{M}$  $= -ig$

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- Highlights

  1) Both CP-even/odd higgs bosons are included
  2) Charged higgs bosons and FCNCs can be treated

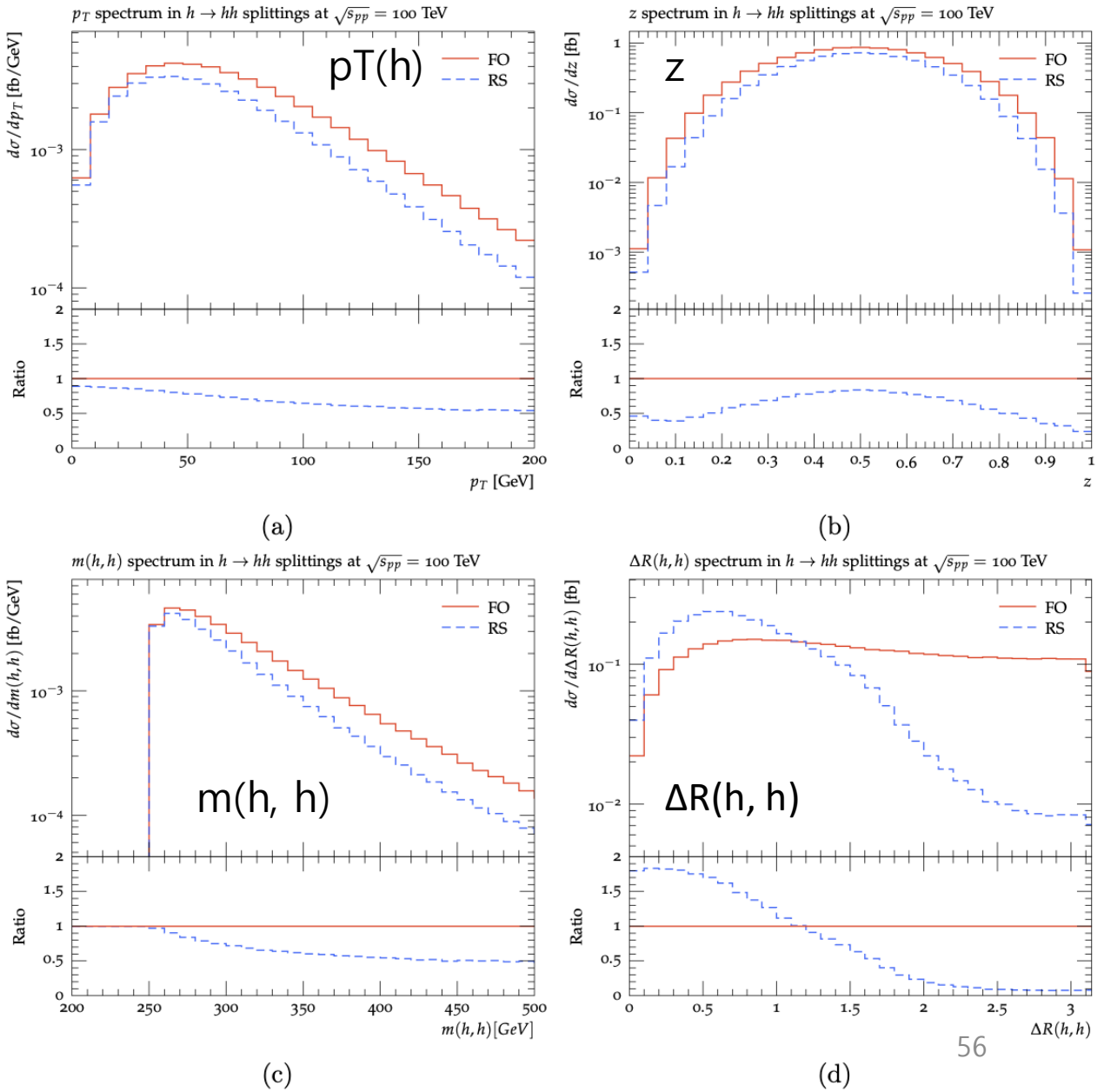
Simplest case: (spin) 0 → 00 splitting

- SM h → hh splitting can be obtained with SM higgs parameters

$$P_{h\to hh}(z,\tilde{q}) = \frac{g_W^2}{z(1-z)\tilde{q}^2}\frac{9m_h^4}{16m_W^2}$$

→ advantage of the generalized parton shower

- Validation
  - 100 TeV pp collider
  - **Resummed-shower (RS)** result
    pp → Zh (MG5) + h → hh (HW7)
  - restriced to have only 1 BSM splitting
  - **Fixed-order (FO)** result
    pp → Zhh (MG5)
  - the second higgs boson should come from the higgs boson (not from Z)



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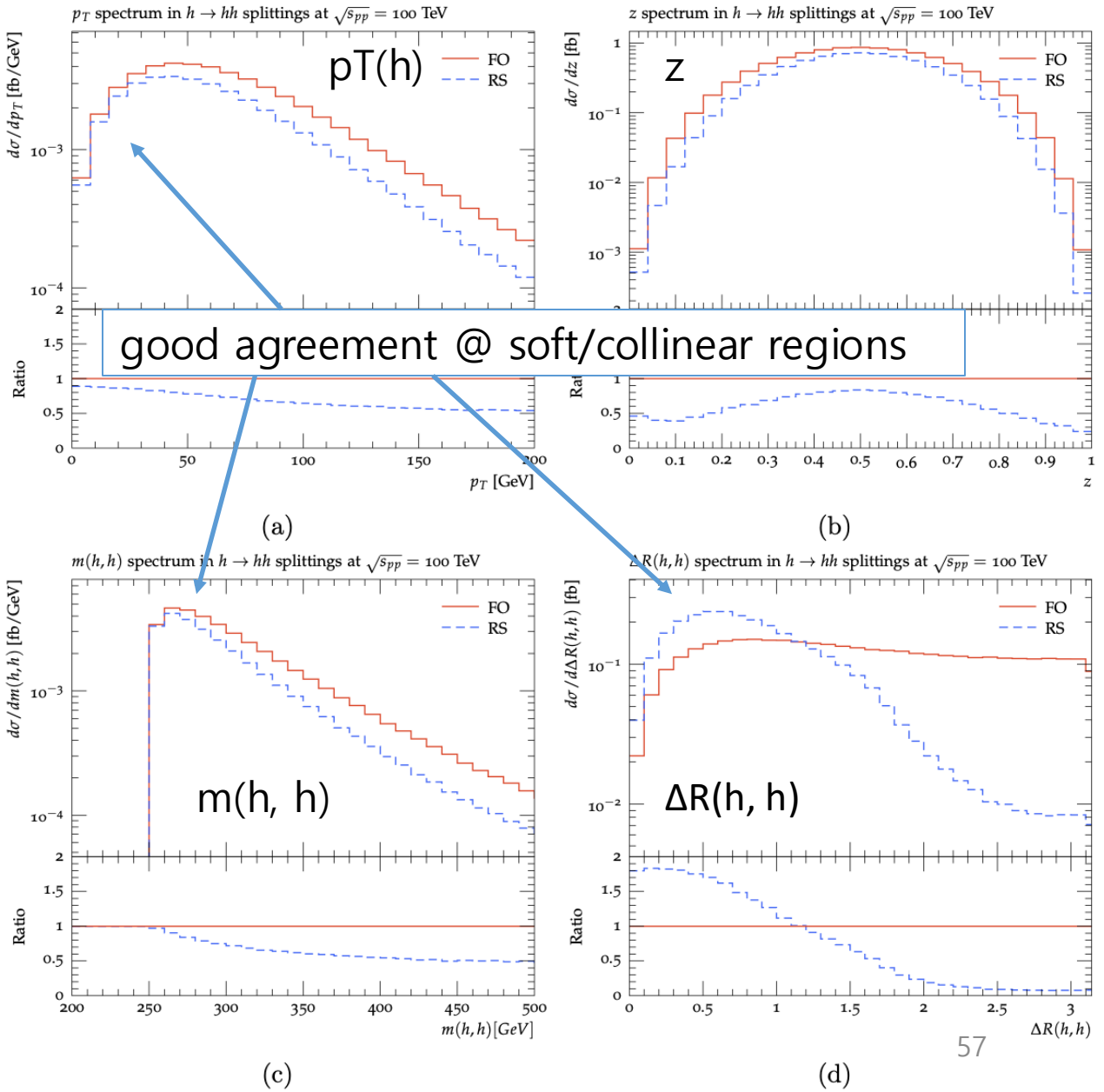

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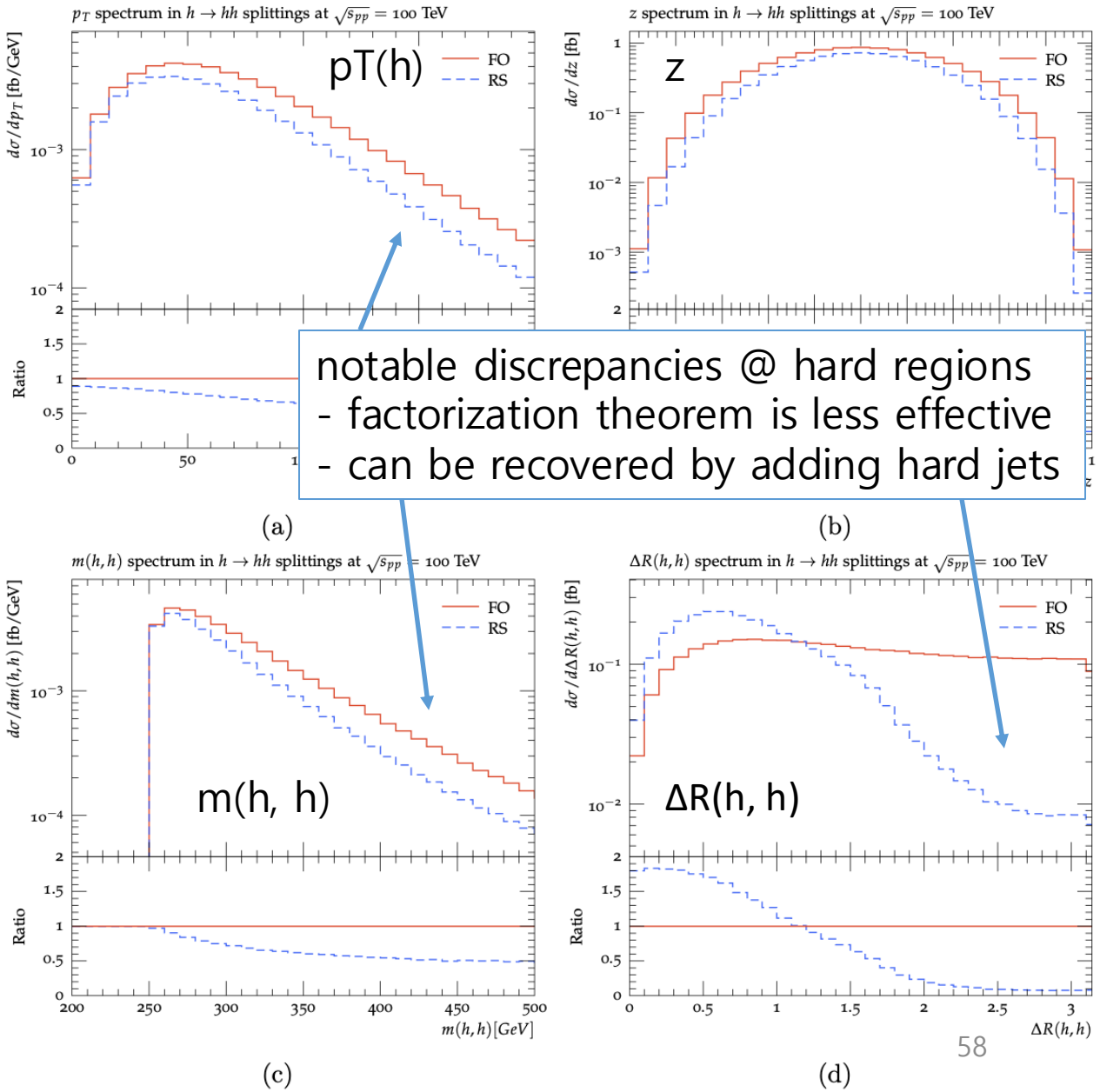

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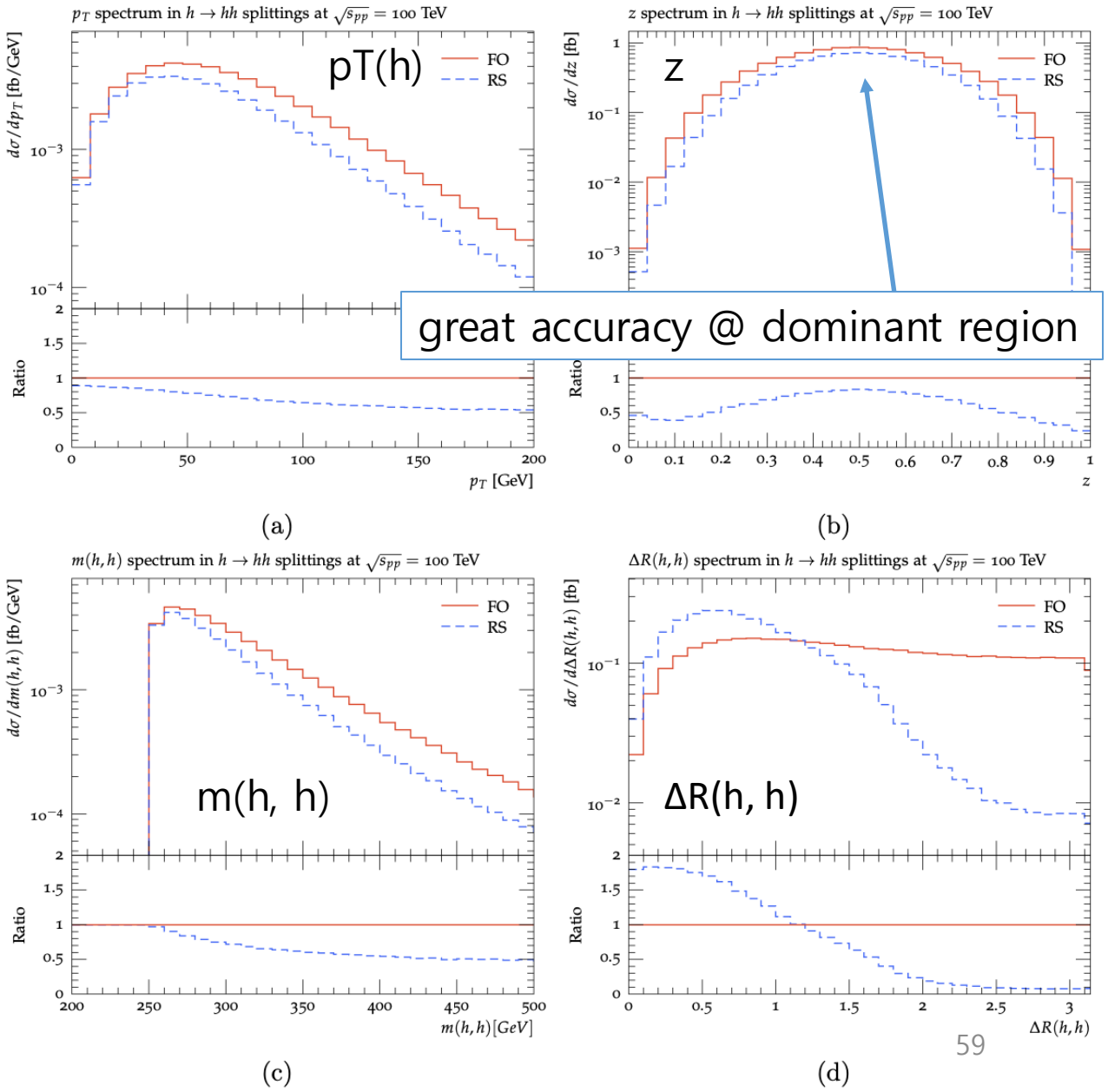
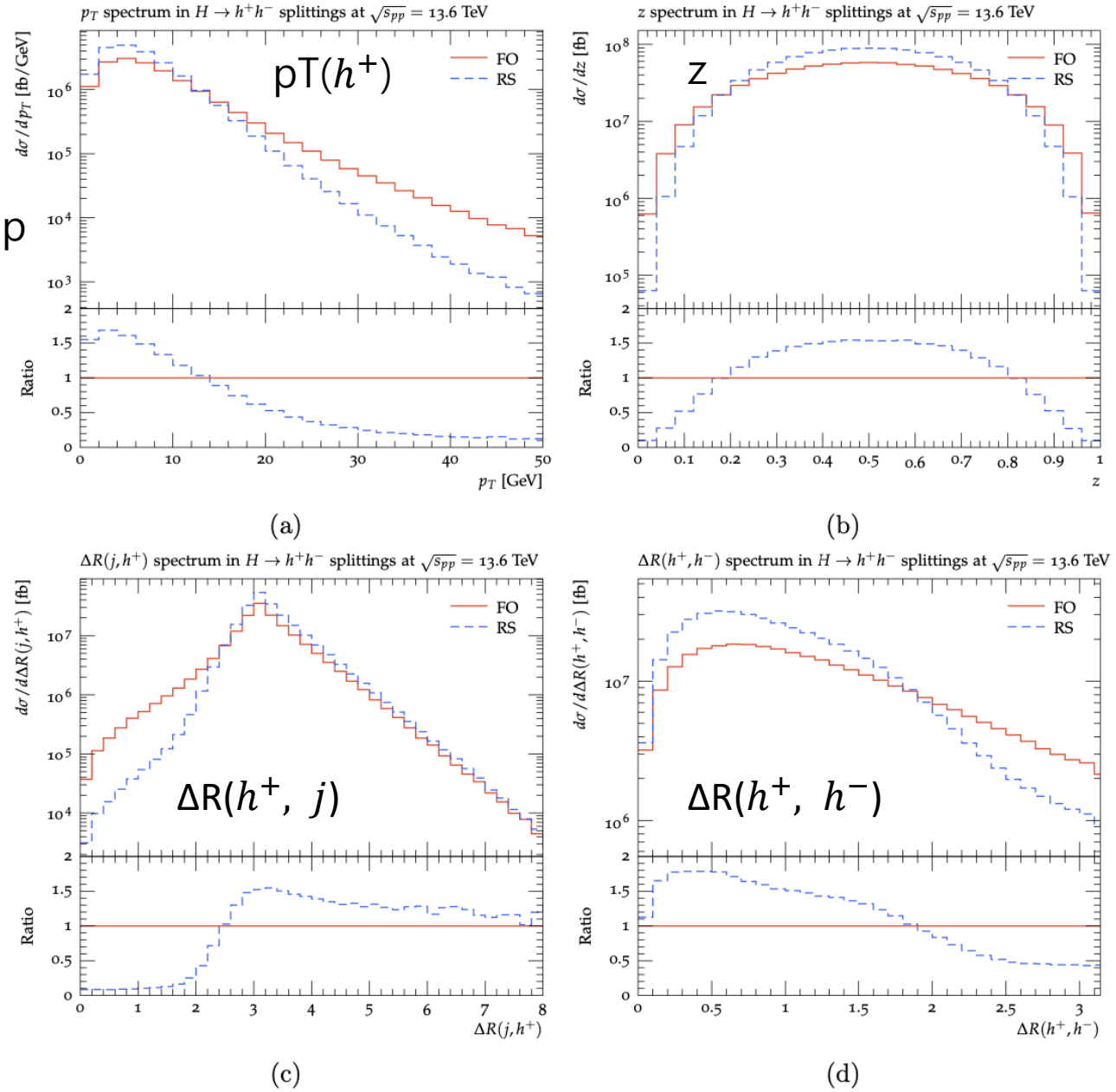


Simplest case: (spin) 0 → 00 splitting

- BSM $\boldsymbol{H \to h^+h^-}$ **splitting** in general 2HDM
  - m(H, h^+) = 10 GeV
- pp → H+j as a baseline @ 13.6 TeV LHC setup
  - **RS: pp → H j (MG5) + H → $\mathbf{h^+h^-}$ (HW7)**
  - **FO: pp → $\mathbf{h^+h^-}$ j (MG5)**
    (All irrelevant diagrams are removed)

- overall shapes are similar to the previous case
  ∵ in both cases $m/\sqrt{s}$ ~ 0.001



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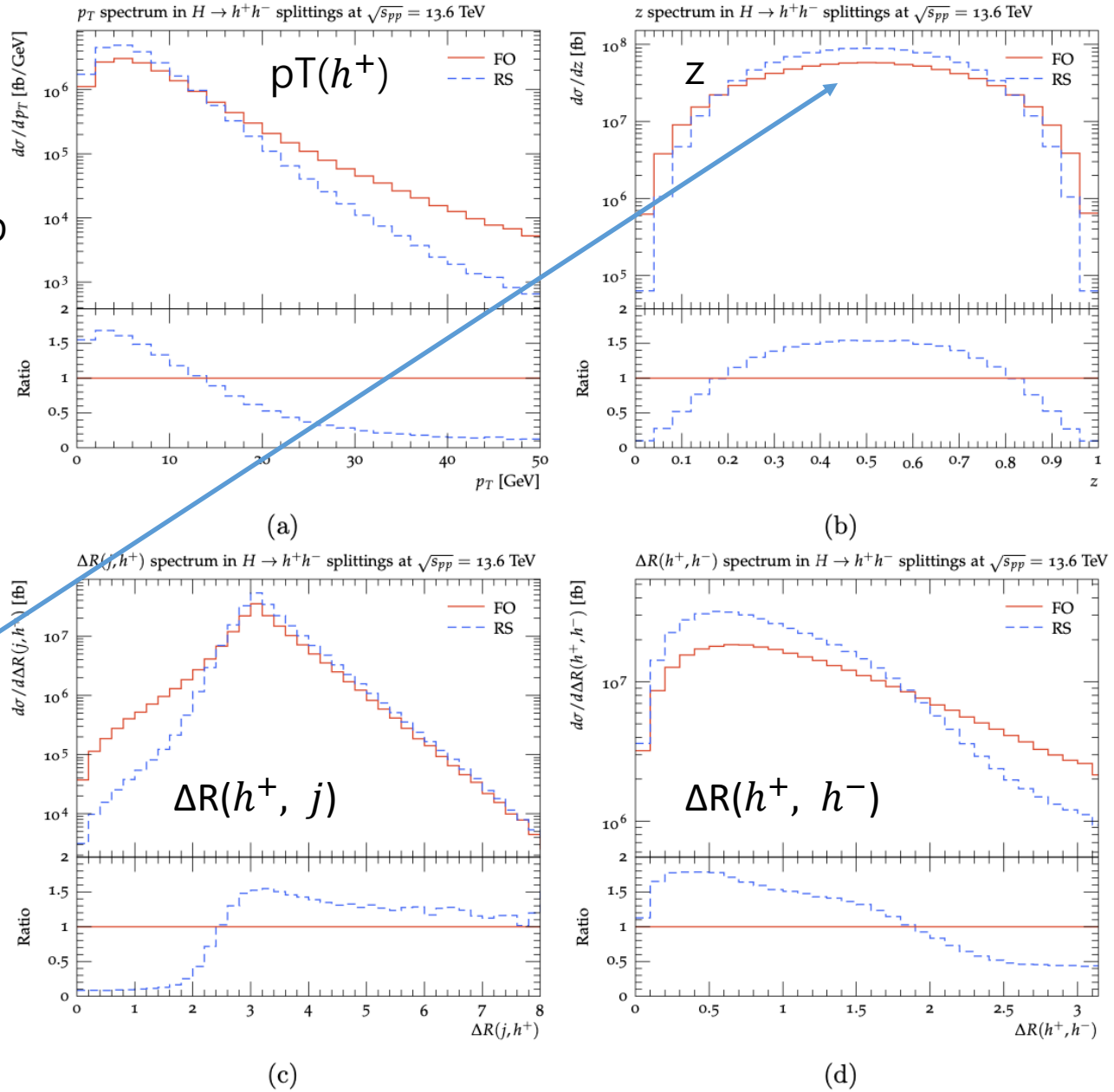
  - **FO: pp → h^+h^- j (MG5)**
    (All irrelevant diagrams are removed)

- overall shapes are similar to the previous case
  ∵ in both cases $m/\sqrt{s}$ ~ 0.001

→ but RS slightly overestimates FO due to the soft/collinear enhancement

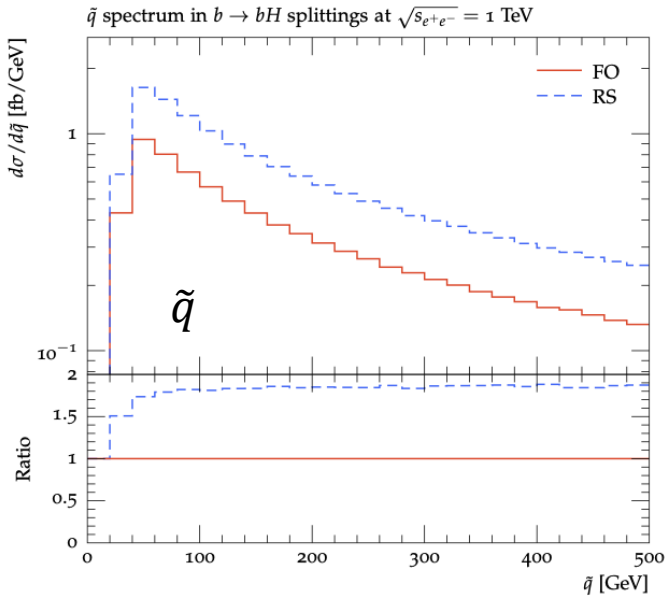


(spin) 1/2 → 1/2 0 splitting

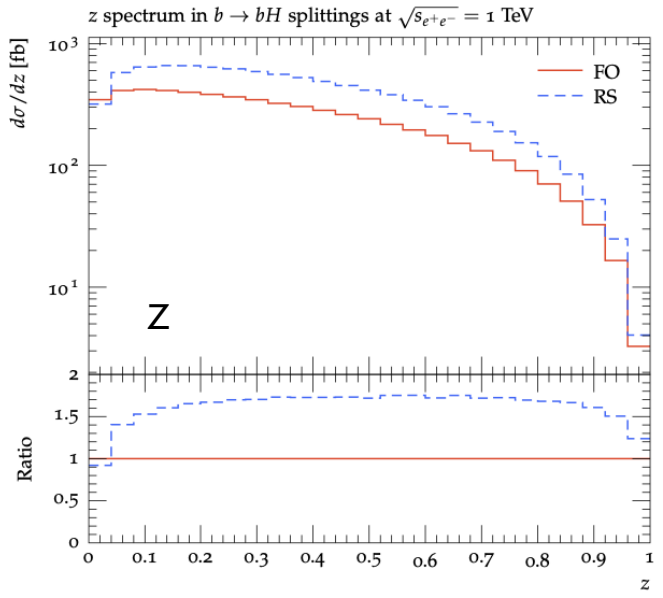
- **b → bH splitting** in general 2HDM
  - m(H) = 10 GeV
- $e^+e^- \to b\bar{b}$ as a baseline @ 1 TeV
  - **RS: $e^+e^- \to b\bar{b}$** (MG5) **+ b → bH** (HW7)
  - **FO: $e^+e^- \to b\bar{b}H$** (MG5)
    (All irrelevant diagrams are removed)

* Shapes are identical

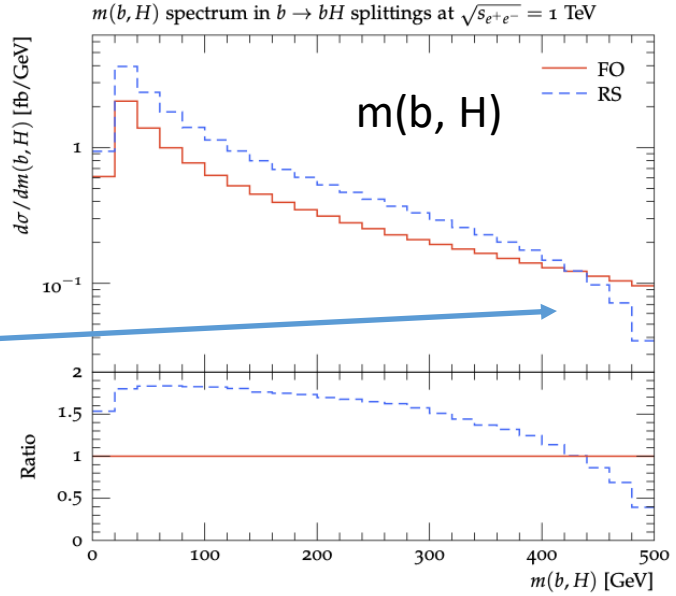
- notable decrease @ m ~ 500 GeV due to centre-of-mass energy limit



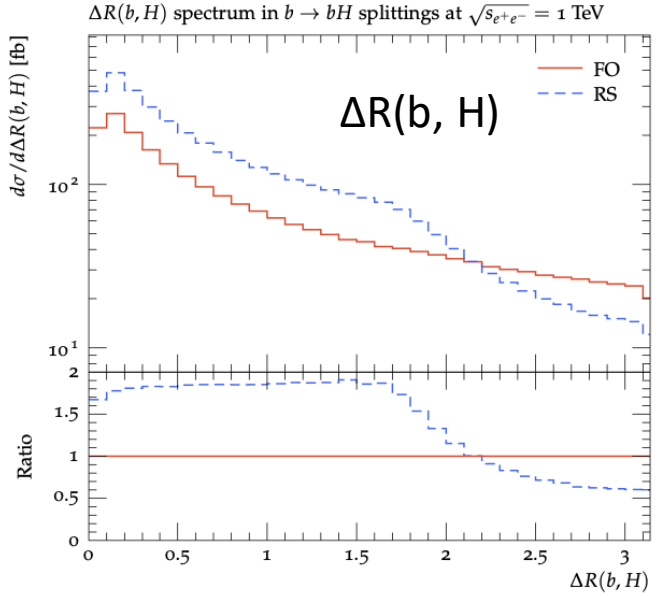
(a)



(b)



(c)



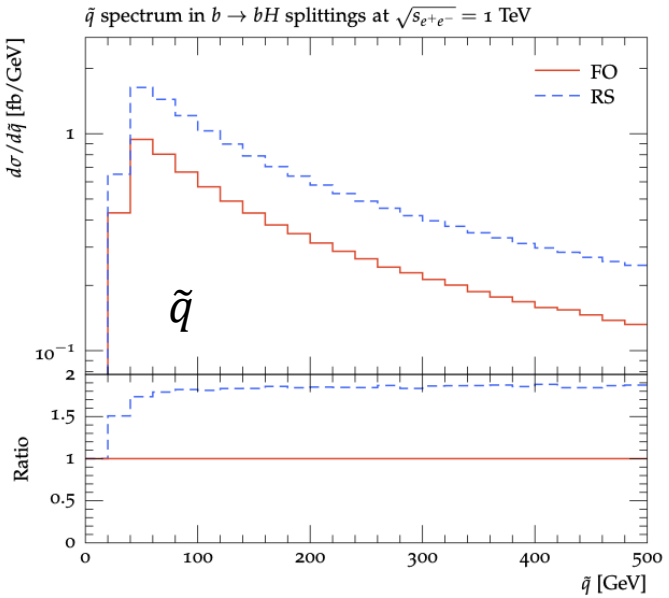
(d)

(spin) 1/2 → 1/2 0 splitting

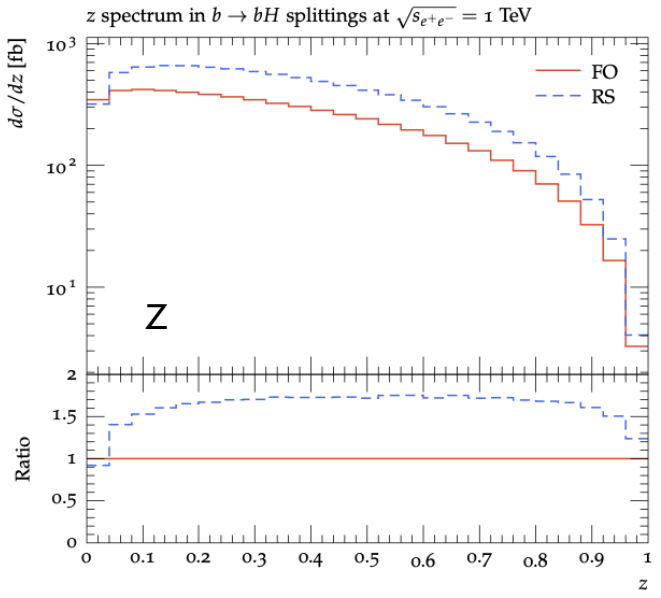
- **b → bH splitting** in general 2HDM
  - m(H) = 10 GeV
- $e^+e^- \to b\bar{b}$ as a baseline @ 1 TeV
  - **RS: $e^+e^- \to b\bar{b}$** (MG5) **+ b → bH** (HW7)
  - **FO: $e^+e^- \to b\bar{b}H$** (MG5)
    (All irrelevant diagrams are removed)

- Overestimation of RS?

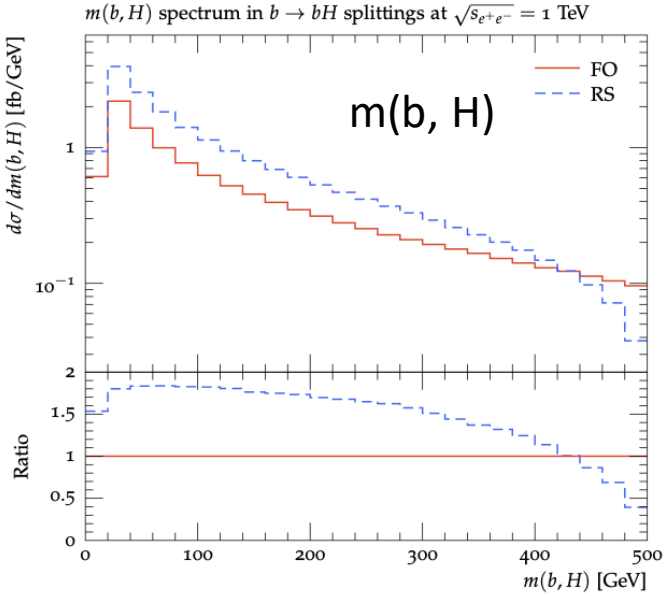
Herwig7 allows dynamial scale choices w.r.t the running b mass @ low scale



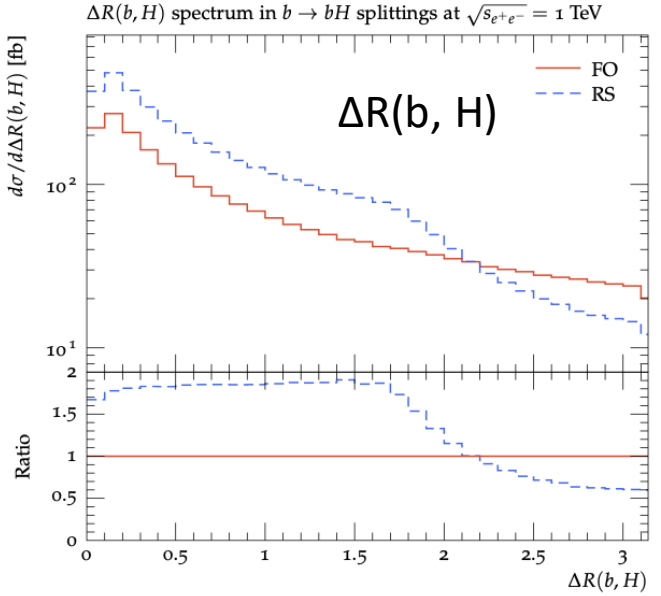
(a)



(b)



(c)



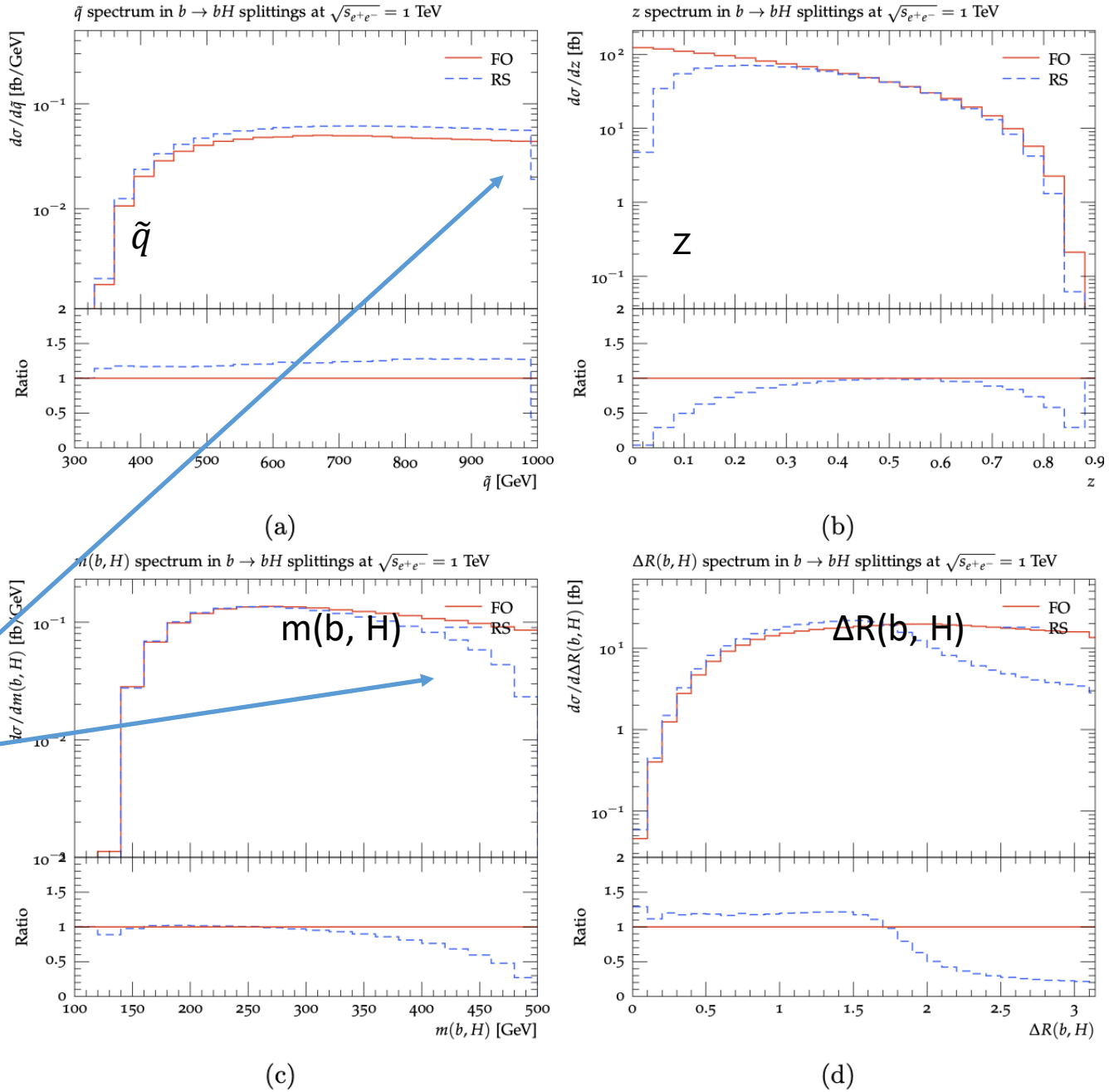
(d)

(spin) 1/2 → 1/2 0 splitting

- **b → bH splitting** in general 2HDM
  - m(H) = 130 GeV
- $e^+e^- \to b\bar{b}$ as a baseline @ 1 TeV
  - **RS: $e^+e^- \to b\bar{b}$** (MG5) **+ b → bH** (HW7)
  - **FO: $e^+e^- \to b\bar{b}H$** (MG5)
    (All irrelevant diagrams are removed)

* Wonderful overall agreements

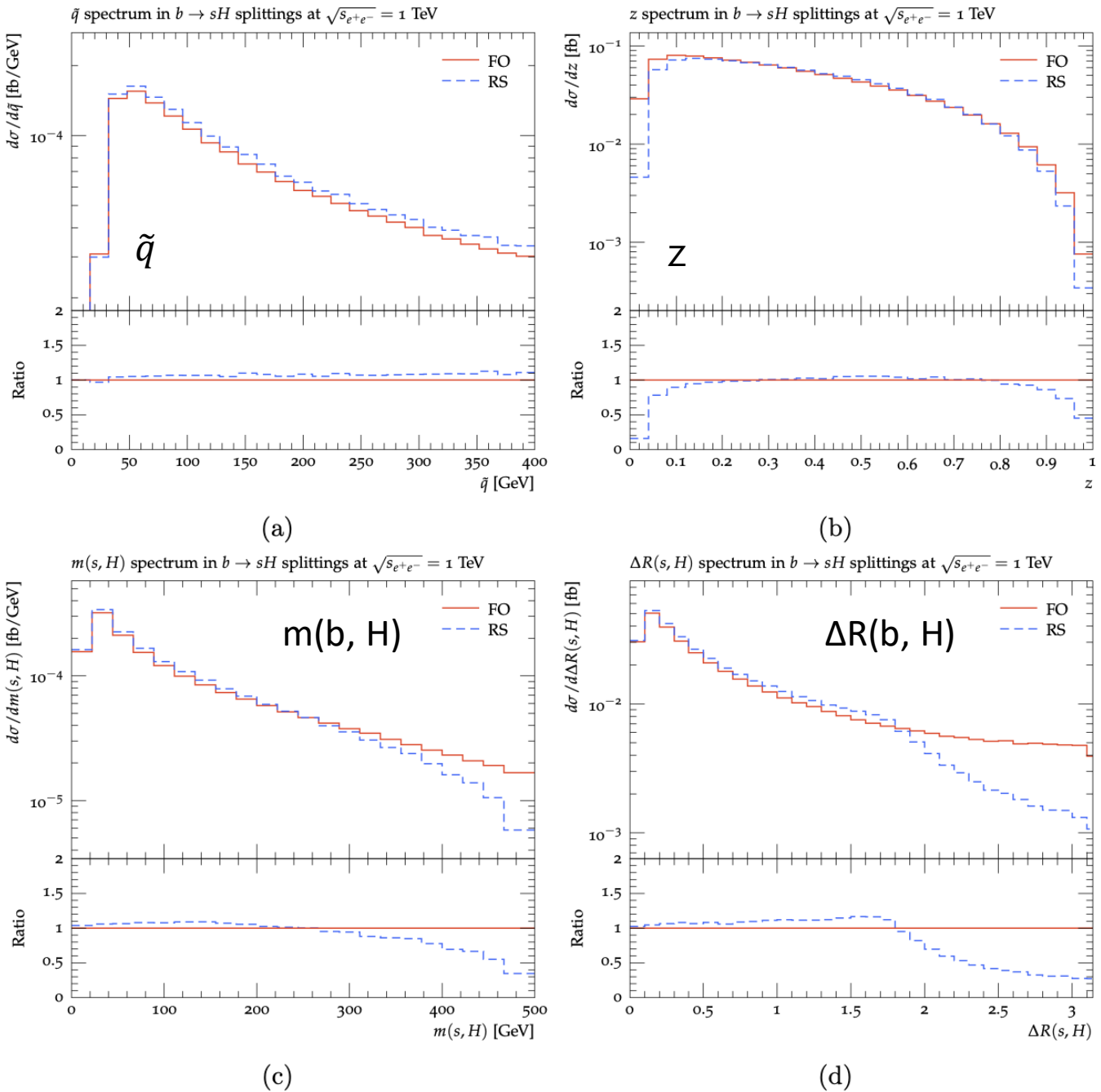
- notable decrease @ m ~ 500 GeV & $\tilde{q}$ = 1 TeV due to $\sqrt{s}$ limit
- also affects to the z distribution



(spin) 1/2 → 1/2 0 splitting

- Important BSM phenomena: **FCNC**!
- **b → sH splitting** in general 2HDM
  - m(H) = 10 GeV
- $e^+e^- \to b\bar{b}$ as a baseline @ 1 TeV
  - **RS: $e^+e^- \to b\bar{b}$** (MG5) **+ b → sH** (HW7)
  - **FO: $e^+e^- \to s\bar{b}H$** (MG5) + counter part
    (All irrelevant diagrams are removed)

* Wonderful overall agreements



A cheating sheet for you

- upto $m^2(p_T^2)$ order in quasi-collinear limit(m, $p_T \to 0$)

1. Splitting out of spin-0 particles

$$P_{f\to f'\phi}(z,\ \tilde{q}) = \frac{1}{2(q_0^2 - m_0^2)}\overline{\sum_{\text{pol}}}|\mathcal{M}|^2$$

$$= \frac{g^2}{2}\Big[\ \left(\rho_+|\kappa+\tilde{\kappa}|^2 + \rho_-|\kappa-\tilde{\kappa}|^2\right)\left[(1-z) - m_{2,t}^2\right] + (\rho_+ + \rho_-)\big[|\kappa|^2(m_{0,t}+m_{1,t})^2$$
$$+ |\tilde{\kappa}|^2(m_{0,t}-m_{1,t})^2\big] + 2(\rho_+ - \rho_-)\Re(\kappa\tilde{\kappa}^*)\left[(1-2z)m_{0,t}^2 + m_{1,t}^2\right]\Big],$$

$$P_{\phi\to\phi'\phi''}(z,\tilde{q}) = \frac{g^2}{2Sz(1-z)\tilde{q}^2}$$

2. Splitting out of spin-1 particles

$$P_{f\to f'V}(z,\tilde{q}) = (|g_R|^2\rho_+ + |g_L|^2\rho_-)\left(\frac{1+z^2}{1-z}(1+m_{0,t}^2) - \frac{1+z}{1-z}m_{1,t}^2 - m_{2,t}^2\right)$$
$$+ (|g_R|^2\rho_- + |g_L|^2\rho_+)zm_{0,t}^2 - 2\Re(g_Lg_R^*)(\rho_+ + \rho_-)m_{0,t}m_{1,t}.$$

$$P_{V\to V'\phi}(z,\tilde{q}) = \frac{g_{BSM}^2}{2}\left[\frac{\rho_+ + \rho_-}{2m_1^2}\Big(z(1-z) + z(1-z)m_{0,t}^2 + (1+z)m_{1,t}^2 - zm_{2,t}^2\Big)\right.$$
$$\left. + \frac{\rho_0}{z^2m_0^2}\Big(z(1-z) + z(1-z)m_{0,t}^2 - (1-z)m_{1,t}^2 - zm_{2,t}^2\Big)\right]$$

$$P_{V\to V'V''}(z,\tilde{q}) = 2g^2\left[\frac{(1-z(1-z))^2}{z(1-z)}(\rho_+ + \rho_-) + 2\rho_0(1-z)^2m_{0,t}^2\right.$$
$$\left. + \frac{(1-z(1-z))^2(m_{0,t}^2) - (1-z^2(1-z))m_{1,t}^2 - (1-z(1-z)^2)m_{2,t}^2}{z(1-z)}(\rho_+ + \rho_-)\right]$$

$$P_{V\to\phi\phi'}(z,\tilde{q}) = g^2\left[(\rho_+ + \rho_-)\left(z(1-z)(1+m_{0,t}^2) - (1-z)m_{1,t}^2 - zm_{2,t}^2\right.\right.$$
$$\left.\left. + \rho_0\frac{(1-2z)^2}{2}m_{0,t}^2\right].\right.$$

$$P_{\phi\to\phi'V}(z,\tilde{q}) = g^2\left[\frac{2z}{1-z}(1+m_{0,t}^2) - \frac{2}{1-z}m_{1,t}^2 + \frac{1}{2}m_{2,t}^2\right]$$

VVV Splitting functions

- upto $m^2(p_T^2)$ order in quasi-collinear limit(m, $p_T \to 0$)

Process	Splitting Function
V→V′V″	$P_{V\to V'V''}(z,\tilde{q}) = 2g^2\left[\frac{(1-z(1-z))^2}{z(1-z)}(\rho_+ + \rho_-) + 2\rho_0(1-z)^2 m_{0,t}^2 + \frac{(1-z(1-z))^2(m_{0,t}^2) - (1-z^2(1-z))m_{1,t}^2 - (1-z(1-z)^2)m_{2,t}^2}{z(1-z)}(\rho_+ + \rho_-)\right]$

- Crossing symmetry [57]:

$$\mathcal{M}^{A\to BC}_{\lambda_0,\lambda_1,\lambda_2}(z,\tilde{q};m_0,m_1,m_2) \propto \mathcal{M}^{A\to CB}_{\lambda_0,\lambda_2,\lambda_1}(1-z,\tilde{q};m_0,m_2,m_1).$$

- Drell-Levy-Yan relation:

$$P^{A\to BC}_{\lambda_A,\lambda_B,\lambda_C}(z) = (-1)^{\lambda_A+\lambda_B+\lambda_C} z P^{B\to AC}_{\lambda_B,\lambda_A,\lambda_C}(1/z).$$

- Parity [34, 35]:

$$\mathcal{M}_{-\lambda_0,-\lambda_1,-\lambda_2} = (-1)^{\lambda_0+\lambda_1+\lambda_2}\mathcal{M}_{\lambda_0,\lambda_1,\lambda_2}.$$

- VVV process is a maximally symmetric process, even though its form looks messy

1→10 spltting function

- We can restore an interesting feature of the goldstone mode in V→Vφ splitting

$$P_{V\to V'\phi}(z,\tilde{q}) = \frac{g_{BSM}^2}{2}\left[\frac{\rho_+ + \rho_-}{2m_1^2}\left(z(1-z) + z(1-z)m_{0,t}^2 + (1+z)m_{1,t}^2 - zm_{2,t}^2\right)\right.$$
$$\left. + \frac{\rho_0}{z^2 m_0^2}\left(z(1-z) + z(1-z)m_{0,t}^2 - (1-z)m_{1,t}^2 - zm_{2,t}^2\right)\right]$$

* massless case
No longitudinal polarization & mass

$$P_{V_{\text{massless}}\to V'_{\text{massless}}\phi}(z,\tilde{q}) = \frac{g_{BSM}^2}{2z(1-z)\tilde{q}^2}$$

[0→00 splitting function]

$$P_{\phi\to\phi'\phi''}(z,\tilde{q}) = \frac{g^2}{2Sz(1-z)\tilde{q}^2}$$

! The goldstone mode in the vector bosons pop up !

Spin-2 Splitting functions (next paper)

- Considering Kaluza-Klein excitations of gravitons

Eur.Phys.J.C56:435-447,2008

Fermion-Fermion-Graviton Lagrangian:

$$\mathcal{L}_{\rm FFT} = 4{\rm GT}\, T^{\mu\nu *}\Big[-\eta_{\mu\nu}\Big\{\bar{f}(i\gamma^\rho\partial_\rho - m_F)f - \frac{1}{2}\partial^\rho(\bar{f}i\gamma_\rho f)\Big\} + \Big\{\frac{1}{2}\bar{f}i\gamma_\mu\partial_\nu f - \frac{1}{4}\partial_\mu(\bar{f}i\gamma_\nu f) + (\mu\leftrightarrow\nu)\Big\}\Big], \quad (28)$$

$$P_{q\to qG} = \rho_+\Bigg[\Big[\frac{4p_T^2}{\sqrt{z}(1-z)^2}\Big]^2 + \Big[\frac{4p_T(m_1 - zm_0)}{(1-z)\sqrt{z}}\Big]^2 + \Big[2p_T\frac{(1-z)^2(m_1+zm_0)m_1 - 2z^2m_2^2 + (1-z)p_T^2}{(1-z)^2z^{3/2}m_2}\Big]^2 + \Big[2\frac{m_1\big(m_1^2(1-z)^2 - z^2m_2^2 + (1+z)p_T^2\big) - zm_0\big((1-z)^2m_1^2 - z^2m_2^2 + 2p_T^2\big)}{(1-z)z^{3/2}m_2}\Big]^2 + \ldots\Bigg] + \rho_-(\ldots)$$

- This is completely new feature. Anyone has never seen this spin-2 splittings! -

~~Why BSM parton shower?~~